Preparation for AP Physics C coming from AP1

All text and problems come from the AP Physics 1 and 2 packets.

<u>Circular Motion: Equilibrium & Torque</u>

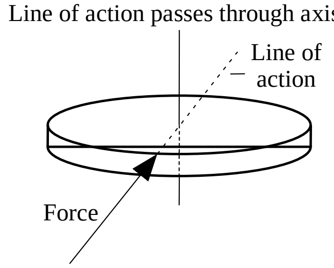
Equilibrium

So far we have used the term *equilibrium* to mean *no net force*; in other words, $\Sigma \vec{F} = 0$. This has been sufficient to describe the equilibrium of objects that can be considered *point masses*, with no extension in space. The only way point particles can move is to *translate* as a whole from one place to another.

Objects with extension in space can also *rotate*, or move in such a way that the points on the object move in circles whose centers form a line called the *axis of rotation*. For an extended object to be at equilibrium, it must be at both translational and rotational equilibrium. The condition $\Sigma \vec{F} = 0$ establishes translational equilibrium, so we need a similar condition that will hold for rotational equilibrium.

Torque

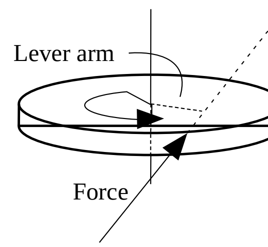
No rotation:
Line of action passes through axis



It takes a force to get an object at rest to rotate, but the force has to be applied in a particular way. If the force is pointed at or away from the axis of rotation, it will not cause rotation no matter how strong it is. The line along which the force acts, called the *line of action* of the force, must "miss" the axis by a certain distance.

A force whose line of action misses the axis is said to cause a *torque* about the axis. The magnitude of the torque depends both on the strength of the force and the distance by which its line of action misses the axis.

Rotation:
Line of action misses axis



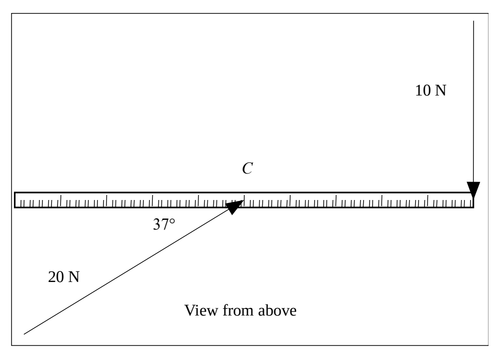
The perpendicular distance from the line of action of the force to the axis of rotation is called the *lever arm* of the force, symbolized by ℓ. The torque caused by a force (symbolized by τ, the Greek letter *tau*) is the product of the magnitude of the force F and the lever arm ℓ:

$$\tau = F \times \ell$$

Torque is measured in Newton meters in the SI system. Torque can be *positive* or *negative*, depending on whether we choose *clockwise* or *counterclockwise* rotation as positive. This choice is arbitrary.

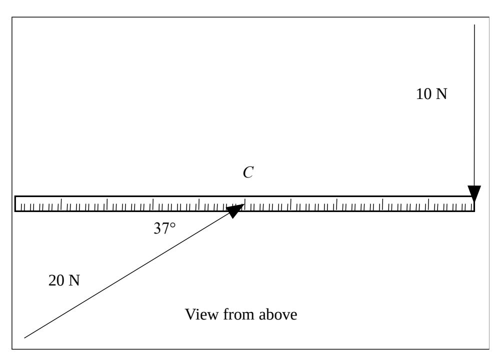
Examples

1. A meter stick lies on a table, and two horizontal forces act on it as shown in the overhead view to the right. The 20 N force acts at the center C of the stick at a 37° angle, and the 10 N force acts perpendicular to the stick at one end.



   A. Choose the left end of the meter stick as the pivot. On the diagram to the right, draw the lever arms of each force, and determine their magnitudes.

   B. Choose *clockwise* as the positive direction for torque. Calculate the torque due to each force, and the *net* torque about the chosen pivot.

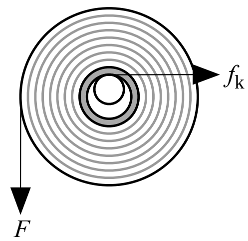


   C. Choose the right end of the meter stick as the pivot. On the diagram to the right, draw the lever arms of each force, and determine their magnitudes.

   D. Choose *clockwise* as the positive direction for torque. Calculate the torque due to each force, and the *net* torque about the chosen pivot.

E. Choose the center of the meter stick as the pivot, and *clockwise* as the positive direction for torque. Calculate the *net* torque about the chosen pivot.

2. A roll of paper towels is shown in cross section to the right. A force F = 3 N pulls a towel downward, while a frictional force f_k = 1 N acts on the inside of the cardboard tube at the center of the roll. If the radius of the roll is 8 cm, and the inside radius of the tube is 2 cm, what is the net torque on the roll about its center?



Equilibrium in Two Dimensions

For an extended object to be in equilibrium, it must be at both translational equilibrium and rotational equilibrium. For an object capable of moving in two dimensions, this condition is equivalent to three equations:

$$\sum F_x = 0, \quad \sum F_y = 0, \text{ and } \sum \tau = 0$$

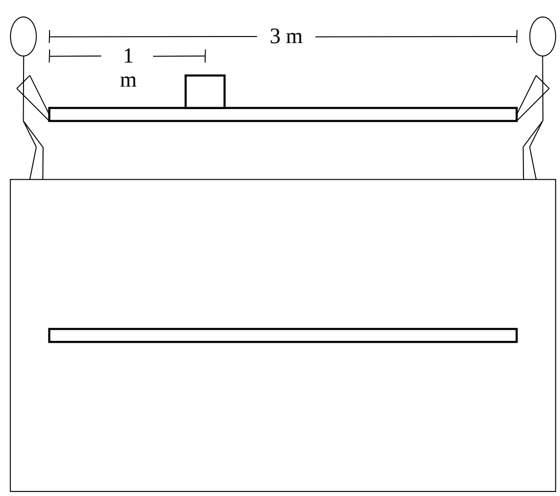
Solving problems in equilibrium involves the same steps as solving $F = ma$ problems, but a is always zero, and the net torque is also zero. Here are the short version of the four steps, with additional notes for equilibrium problems:

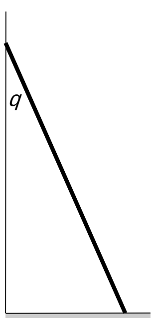
1. Isolate the system
2. Draw an *extended* free-body diagram (weight, surfaces, strings with forces drawn at their point of application. The weight force acts at the *center of mass*, which for a symmetric object is at its geometric center.)
3. Choose axes *and a pivot* and resolve (the choice of axes is arbitrary because a = 0).
4. Apply $\sum F_x = 0$, $\sum F_y = 0$, and $\sum \tau = 0$ as needed.

Examples

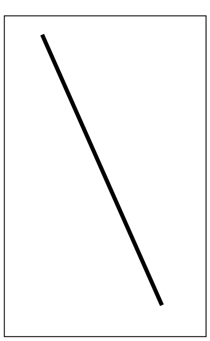
1. Two men carry a heavy 10 kg plank 3 m long with a 15 kg box placed 1 m from one end of the plank. The men carry the plank by applying vertical forces at each end.
   A. In the space provided, draw an extended free-body diagram of the plank. Clearly show your choice of pivot.

   B. Determine the vertical forces exerted by each man.





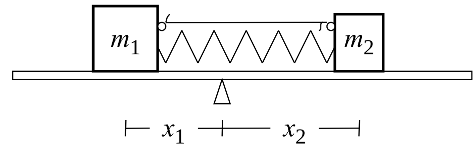
2. A uniform stick of length L and mass m leans against a frictionless wall at an angle θ with the vertical, as shown to the left.
   A. In the space provided, draw an extended free-body diagram of the stick. Clearly show your choice of pivot.



   B. Find the minimum coefficient of static friction between the stick and the floor so that the stick will not slip.

   C. Does your answer makes sense when θ = 0 and θ = 90°? Explain.

Center of Mass



The fulcrum in the image at right is under a point called the *center of mass* of the two blocks. In general, if there is no net external force on a system of objects, the velocity of the center of mass remains constant; in this case, zero. The center of mass is in a sense the *average location of the mass* of a system. It is a *weighted average*, weighted according to the mass of each object in the system.

To calculate the position of the center of mass of a system of two or more masses in one dimension, we multiply (weight) the position of each object by the mass, add those values up, and divide by the total mass:

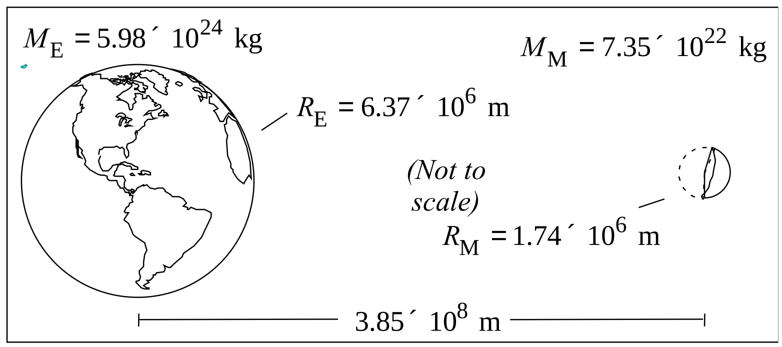
$$x_{\mathrm{cm}} = \frac{m_1 x_1 + m_2 x_2 + \cdots}{m_1 + m_2 + \cdots}$$

Similarly one can calculate the velocity of the center of mass of a system by calculating the weighted average velocity:

$$v_{\mathrm{cm}} = \frac{m_1 v_1 + m_2 v_2 + \cdots}{m_1 + m_2 + \cdots}$$

Example

The masses and radii of the earth and moon are shown to the right, along with the average distance between their centers.



A. On a scale where the earth is represented by a 12-inch globe (with a diameter of 12 inches), what would the diameter of the moon be?

B. On that scale, what would the earth-moon distance be in feet?

C. Find the distance from the center of the earth to the center of mass of the earth-moon system. Using the radius of the earth as a basis, mark the approximate location of the center of mass on the drawing of the earth above.

Conservation of Momentum in Two Dimensions

Impulse and momentum are vector quantities, and the conservation of momentum is valid in two or three dimensions as well as one: If there is no net external force on a system, the momentum vector for the system will remain the same in magnitude and direction. The momentum vector for a system corresponds to the momentum of the *center of mass of the system*.

Example

A Toyota Tercel going makai on Pensacola St. approaches the intersection with Kapiolani at 35 mph (15 m/s). A Ford F150 pickup going Ewa on Kapiolani fails to stop at the light and crashes into the Toyota (the collision is at right angles). The police note in the accident report that the tire marks of the wreckage are at a 37° angle with the original direction of the truck. The masses of the two vehicles are 1,045 kg and 2180 kg, respectively.

A. What was the makai component of the momentum of the Toyota just before the collision?

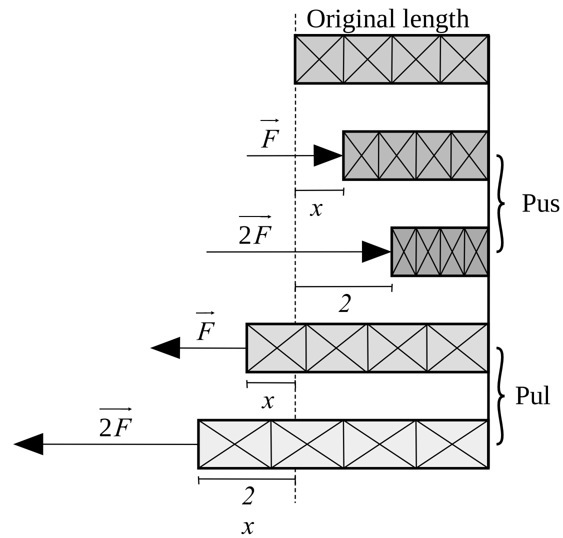
B. What was the makai component of the velocity of the wreckage just after the collision?

C. What was the Ewa component of the velocity of the wreckage just after the collision?

D. What was the initial speed of the truck?

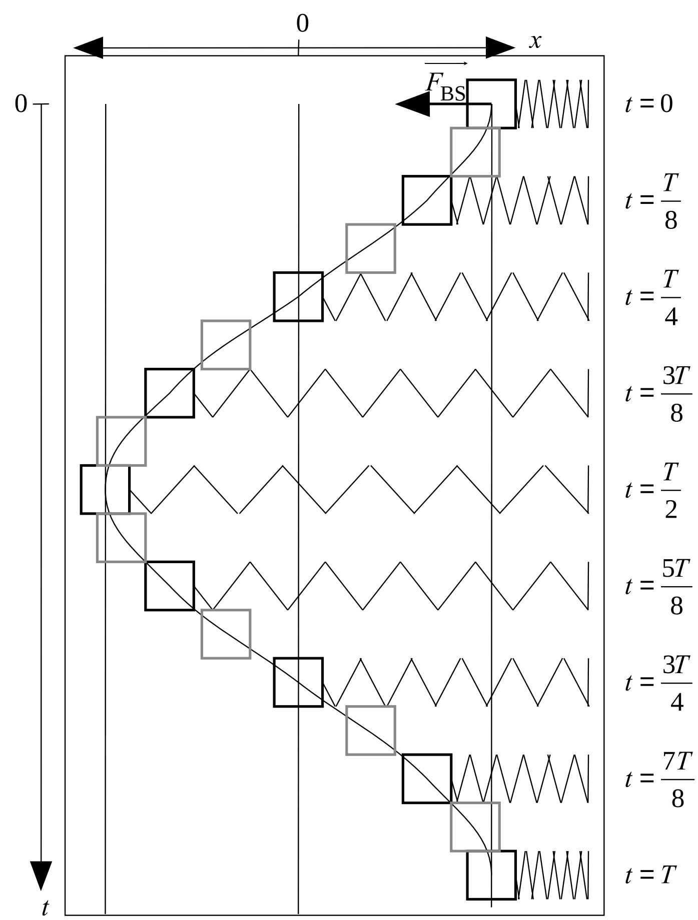
Elasticity and Hooke's Law

In Unit 6, the word *elastic* was used in connection with collisions, to describe those in which kinetic energy is conserved. In this chapter we use it in another, related sense: Many materials have the property that they can be stretched or compressed and will return to their original shape; these are called *elastic* materials. Often, the amount of stretch or compression these materials undergo is directly proportional to the force applied to them. This observation is known as *Hooke's Law* after Robert Hooke, who first formulated it. In what follows, we will assume that elastic materials obey Hooke's Law.



If we $\left\{\begin{matrix}\text{push}\\ \text{pull}\end{matrix}\right\}$ on a piece of elastic material with a force $\vec{F}$, and as a result it $\left\{\begin{matrix}\text{compresses}\\ \text{stretches}\end{matrix}\right\}$ a distance x, then Hooke's Law states that if we double the force to $\overrightarrow{2F}$, then the $\left\{\begin{matrix}\text{compression}\\ \text{stretch}\end{matrix}\right\}$ doubles to $2x$. In general, we can write that in equation form as $F = kx$, where F is the magnitude of $\vec{F}$, and k is a proportionality constant, often called the *elastic constant* or the *spring constant* because springs are most frequently used as an example of an elastic material. In terms of Newtons (N) and meters (m), what are the SI units of k?

Simple Harmonic Motion



Any motion that repeats itself after a certain time interval is said to be *periodic* or *harmonic*. The *period T* of the motion is the time interval between repetitions of the motion. *Harmonic* refers to musical sound, which is produced by periodic waves caused by vibrations. Harmonic motion of an object requires a *restoring force* to return the object to its *equilibrium position*. When the restoring force obeys Hooke's Law ($\vec{F} = -k\vec{x}$), the motion is said to be *simple harmonic motion*.

The diagram to the right shows one complete cycle of motion for a mass connected to an ideal spring. The mass is pushed aside a distance *A* and released from rest. It moves left through equilibrium, stops, and returns back to the right, where the cycle is repeated.

The locations where the mass stops are called *extremes*, and the distance from the equilibrium position to either extreme is called the *amplitude* of the motion.

The graph traces out a path that looks like the *sine* or *cosine* function, so the motion is called *sinusoidal*.

When a spring is stretched or compressed, it exerts a force and is therefore capable of doing work. The spring is storing a type of potential energy called *elastic potential energy*, PE_{elastic}. The work done to stretch or compress a spring is equal to the elastic potential energy it stores.

We can now describe simple harmonic motion in terms of work and energy. Consider the mass *m* on the end of a spring whose stiffness constant is *k*. We start by pushing the mass from its equilibrium position to a distance *A*, compressing the spring. In doing so, we have done work, which is stored as PE_{elastic} in the spring. As it passes through equilibrium, the box has speed v_{max}.

Examples

1. A spring having a spring constant 250 N/m is connected to a mass of 10 kg. The mass is displaced 30 cm from equilibrium and released.
   A. Find the period of vibration

   B. Find the speed of the mass as it passes through equilibrium

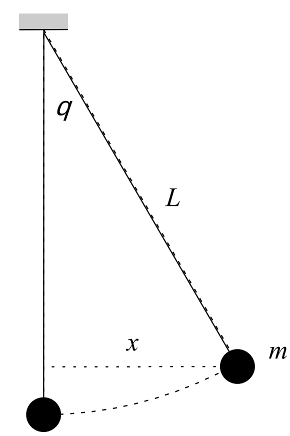
   C. What is the total mechanical energy of the system?

The springs of a 1000 kg car compress vertically 7 mm when a 100 kg man steps in.
   A. What is the effective spring constant of the car's suspension?

   B. With the man in the car, how many vibrations per second does the body of the car make after being jarred while going over a speed bump? (In a real car, the *shock absorber* provides friction so that the vibrations quickly die out.)

The Pendulum

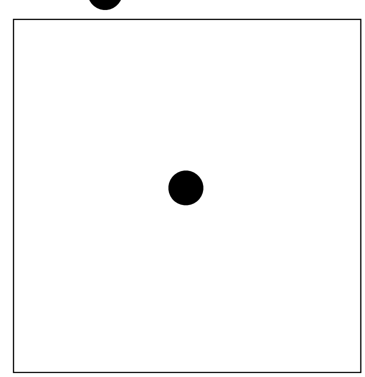
Simple harmonic motion does not necessarily involve springs. For an object to be moving in simple harmonic motion, there must be a restoring force that is proportional to the displacement from equilibrium, by Hooke's Law ($F = -kx$). The constant k is a proportionality constant, not necessarily associated with a spring.



Consider a pendulum, which is a small mass m on the end of a string of length L. We pull the mass aside so that the string makes an angle q with the vertical, and release it from rest. The pendulum will swing back and forth in periodic motion, but in order to show that the motion is simple harmonic, we need to show that there is a restoring force that is proportional to the displacement.

We are going to do this, but only in approximation. The displacement of the mass from equilibrium is really the length of the arc in the diagram to the right, but for small angles this arc length is approximately equal to the distance labeled x in the diagram.

   A. In the space to the right, draw a free body diagram of the mass at the angle q. Resolve mg into components parallel and perpendicular to the string. The component perpendicular to the string is the restoring force F (it points back toward the equilibrium position).



   B. What is sin q in terms of x and L?

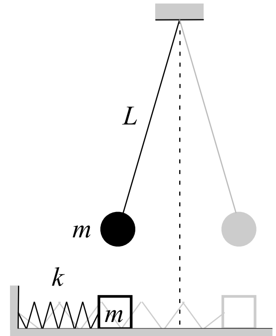
   C. Show that the restoring force F is proportional to the approximate displacement x.

   D. Substitute this proportionality constant into your expression for the period T of any simple harmonic motion, to find the period of a pendulum. (Remember that this expression is only an approximation when the maximum angle q is small.)

E. A pendulum with mass m and length L is arranged to have the same amplitude and period as a mass-spring system with the same mass and spring constant k. If we then double the mass of each to $2m$,

a. By what factor, if any, is the period of the pendulum changed? Explain.

b. By what factor, if any, is the period of the mass-spring system changed? Explain.



Quick Calculus

Find the derivative:

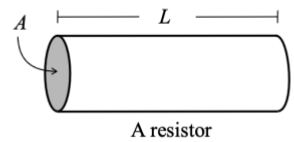
1. $f(x) = 12\pi$
2. $f(x) = 3x + 7$
3. $f(x) = 2x^2 + 3$
4. $f(x) = 10x^3 + 4x$
5. $f(x) = \sin(x)$
6. $f(x) = 3\sin(3x)$
7. $f(x) = 4(x^2 - 1)$
8. $f(x) = 3(x^2 - 1)^2$
9. $f(x) = e^x$
10. $f(x) = \ln x$

Find the integral (ignore C):

11. $f(x) = 23$
12. $f(x) = 6x$
13. $f(x) = \sin(x)$
14. $f(x) = \frac{1}{x}$
15. $f(x) = x^{-2}$
16. $f(x) = (8 - x)^2$

Second Semester

Resistivity



Resistors come in many shapes and sizes, and are made out of different materials. For a given material, the resistance depends only on two things: The length of the resistor (L) and the cross-sectional area (A). The series and parallel rules for resistance tell us how these factors influence the resistance:

A) Use the series rule for resistors to explain how the resistance depends on L.

B) Use the parallel rule for resistors to explain how the resistance depends on A.

The proportionality constant is a property of the material of which the resistor is made. It's called the *resistivity* of the material, and it's symbolized by the Greek letter *rho* (ρ).

C) Write the equation for resistance R in terms of ρ, L, and A:

D) What are the units of resistivity?

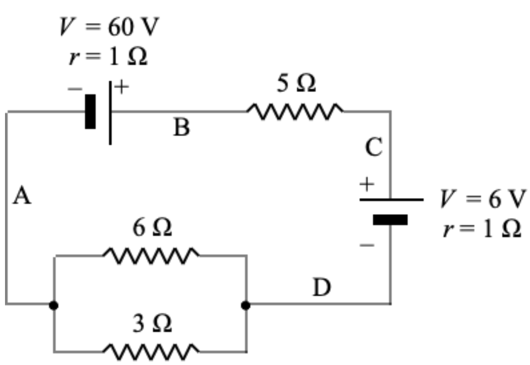
Remember: *Resistance* is a property of an individual resistor. *Resistivity* is a property of matter in general.

Kirchhoff's Rules

There are two simple rules that are followed by the current in any circuit, such as the one in the example above. They are named after Gustav Kirchhoff, a German physicist:

Kirchhoff's Loop Rule: The total change in electrical potential around any closed loop in a circuit is zero.

This rule is a consequence of the conservation of energy; if charges had a net energy gain around a closed loop, they would be continuously gaining energy without spending it.



A) Four points on the circuit in the example above are labeled A - D. For each of the following, determine the potential difference between the points and state which point is at the higher potential:

i) A and B:

ii) B and C:

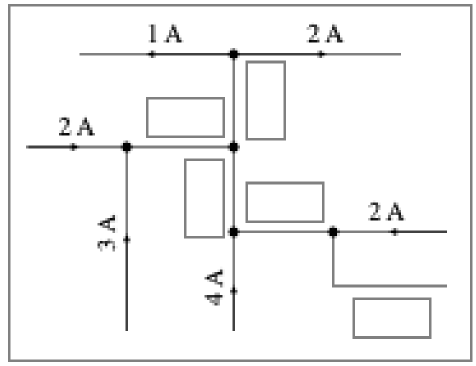
iii) C and D:

iv) D and A:

Kirchhoff's Junction Rule: The total current entering a junction is equal to the total current leaving that junction.
This rule is based on conservation of charge; if the incoming current were greater than the outgoing current, charge would be accumulating at the junction.

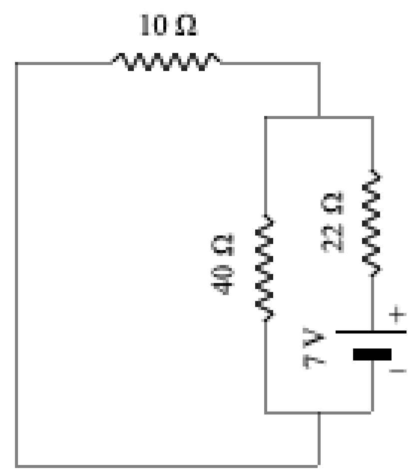
B) There are two junctions in the circuit on the previous page. Explain how Kirchhoff's Junction Rule is satisfied at those junctions.

C) The figure to the right shows a portion of a circuit. Indicate by arrowheads the direction of the current in the unlabeled parts of the circuit, and indicate the magnitude of the current in each.



A 7 V ideal battery is connected to three resistors as shown to the right.

A) Which resistor is in parallel with the 40 Ω resistor? Explain.



B) Find the current in the battery.

C) Find the potential difference across the 22 Ω resistor.

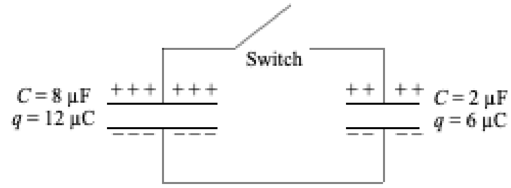
D) Find the current in the 10 Ω resistor.

7. Three capacitors (4.0, 6.0, and 12.0 μF) are connected in series across a 50 V ideal battery.

A) Sketch the circuit in the space to the right.

B) Find the voltage across the 4.0 μF capacitor.

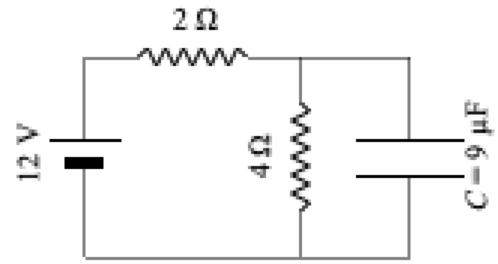
8. The drawing shows two fully charged capacitors with capacitances and charges as shown. The switch is closed, and charge flows until equilibrium is established (i.e., until both capacitors have the same voltage across their plates). Find



A) the resulting voltage across either capacitor

B) the charge on each capacitor

9. The circuit in the drawing shows two resistors, a capacitor, and a battery. When the capacitor is fully charged,

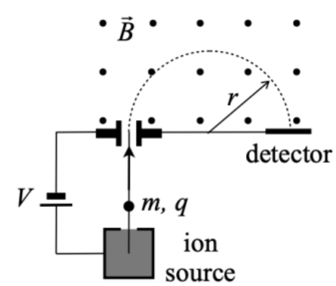


A) what is the current through the 4 Ω resistor?

B) what is the voltage across the 4 Ω resistor?

C) what is the magnitude q of the charge on one of the capacitor plates?

(The Mass Spectrometer) Ions of mass m and charge $+q$ are accelerated through a potential difference V as shown to the right. They then pass into a region of known magnetic field B! and go in a semicircular path, striking a detector.



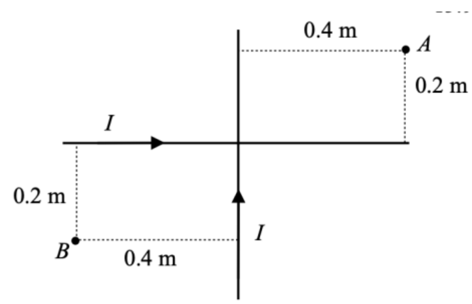
A) What is the kinetic energy of the charge after passing through the potential difference? (Hint: Think of the units of potential difference.)

B) The mass spectrometer is designed to measure the mass of the ion when the magnitude *B* of the magnetic field and the accelerating potential *V* are known. The point where the ion strikes the detector indicates the radius of the ion's path in the magnetic field. If the particle is observed to go in a circle of radius *r*, find the mass in terms of *r*, *B*, *q*, and *V*.

1. Lightning strikes a vertical metal flagpole, and there is a momentary (conventional) current up the pole. What is the direction of the magnetic field due to this current at a point just east of the center of the pole? Explain how you use *RHR#2*.

2. What is the strength of the magnetic field at a point 10 cm away from a long wire in which the current is 20 A?

The drawing to the right shows two perpendicular wires that lie in the plane of the paper. Each wire carries a current $I = 5$ A.



A) Determine the magnitude of the magnetic field at point *A* due to the horizontal wire.

B) Is the direction of the magnetic field at point *A* due to the horizontal wire *up*, *down*, *left*, *right*, *into the page*, or *out of the page*?

C) Determine the magnitude of the magnetic field at point *A* due to the vertical wire.

4. D) Is the direction of the magnetic field at point *A* due to the vertical wire *up*, *down*, *left*, *right*, *into the page*, or *out of the page*?

5. E) What is the magnitude and direction of the net magnetic field at point *A*?

Electromagnetic Induction

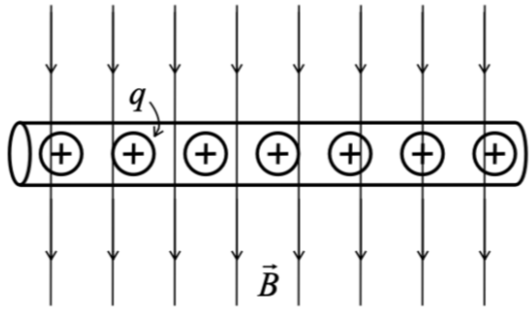
We have seen that a wire can be thought of as a hollow tube filled with positive charges that are free to move within the wire. A current in the wire occurs when the charges flow, and this can result in a magnetic force on the wire.

Now consider the opposite: We exert a force on the wire causing it (and all the positive charges inside) to move through a magnetic field. The resulting magnetic forces on the charges in the wire can give rise to a current. Whether or not a current actually flows depends on whether the wire is part of a complete circuit.

A) If the wire in the diagram to the right moves toward you (out of the page), what is the direction of the force on the positive charges in the wire? Explain your use of *RHR#1*.

q

13.11

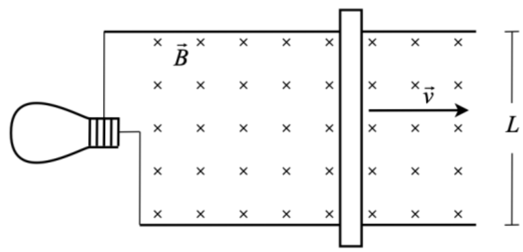


B!

2. B) If the wire moves into the page, what is the direction of the force on the positive charges in the wire?

3. C) If the wire is moved up toward the top of the page, what is the direction of the force on the charges? Explain.

1. A light bulb whose resistance is 96 Ω is connected to two conducting rails a distance $L = 1.6$ m apart. A conducting rod is moved across the rails and perpendicular to a 0.80 T magnetic field at a speed of 5 m/s. The rod and rails have negligible resistance. The magnetic field is directed into the page as shown, and the rod is moved to the right.



A) What is the induced emf in the rod?

B) What is the induced current in the circuit?

C) What is the direction of the induced current in the rod? Explain using *RHR#1*.

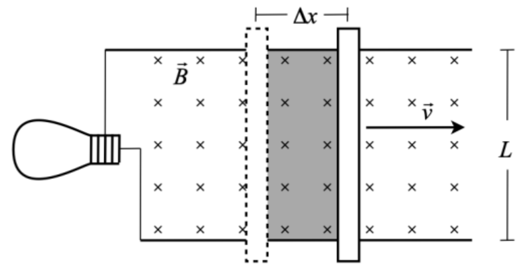
D) What is the electrical power delivered to the bulb?

E) How much energy is used by the bulb in one minute?

F) How much force is needed to keep the rod moving at 5 m/s? Explain why a force is required.

G) How much work is done by the force in one minute? How does this compare with your answer to part E)?

The rod in the example from page 13.12 moves a distance Δx in a time Δt.



A) What is the change in flux $\Delta\Phi$ during this time?

B) Use Faraday's Law to find the induced emf.