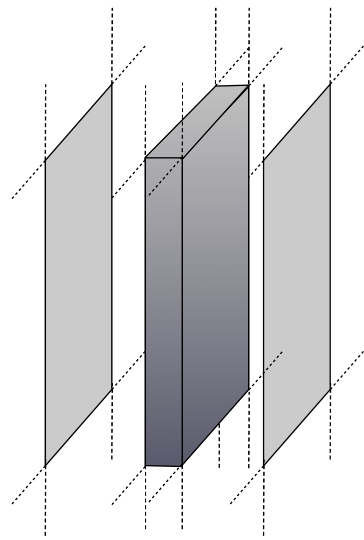

AP Physics C Electricity & Magnetism by Tutorial

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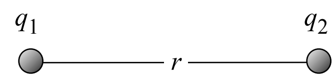
AP Physics C
Unit 9 Name______________________________

Electricity and Magnetism

Our understanding of electrical forces began with the Greeks, who noticed that pieces of amber could be rubbed with cloth, and they would then attract bits of straw. The word *electron* is derived from the Greek word for amber. Magnetic forces were thought of as completely different phenomena until 1820, when Hans Christian Oersted discovered a connection. Subsequently Michael Faraday laid the experimental groundwork and James Maxwell worked out the mathematical theory for what became known as Electromagnetism.

Electrostatics



Benjamin Franklin was one of the first to hypothesize that electric forces are due to a property of matter called *electric charge*, which exists in two varieties: positive and negative. Opposite charges attract each other, and like charges repel, with a force described by *Coulomb's Law*: If q_1 and q_2 are two point charges and r is the distance between them, then they attract or repel (according to their signs) with a force $F = \dfrac{k|q_1||q_2|}{r^2}$, where k is a proportionality constant. The absolute value symbols are usually understood, but are there to emphasize the fact that the signs of the charges are not to be considered algebraic signs.

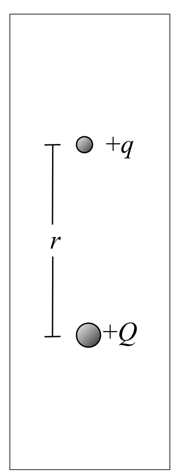
A common unit of electric charge is the *elementary charge e*. This is the charge carried by an electron (negative) or a proton (positive). The elementary charge is the smallest known amount of charge carried by a free particle, and quantities of electric charge are always multiples of the elementary charge. Quarks, which are always found in groups of two or three, have charges of $\pm\frac{1}{3}e$ or $\pm\frac{2}{3}e$, but the allowed groupings always have a net charge that is a whole number multiple of e. The SI unit of electric charge is the *Coulomb* (C), which is $6.25\times10^{18}e$. Therefore the elementary charge in Coulombs is

$e =$ C. The proportionality constant in Coulomb's Law is easy to remember: $k = 9\times10^9\ \dfrac{\text{Nm}^2}{\text{C}^2}$.

Examples

1. The electric force is considerably stronger than the gravitational force. Given that the mass of an electron is $9.11\times10^{-31}\text{kg}$, find the ratio of the electric force to the gravitational force between two electrons a given distance apart.
 ($G = 6.67\times10^{-11}\text{Nm}^2/\text{kg}^2$)

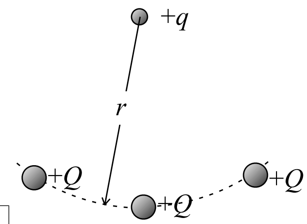
2. A small positive charge $+q$ and a larger positive charge $+Q$ are held a distance r apart.
 A) On the drawing to the right, draw vectors indicating the electric forces exerted on each charge by the other.
 B) Which of these forces, if either, is greater? Explain.
 C) By what factor would the force on the $+q$ charge change if the charges were moved to a separation distance of $2r$? Explain.



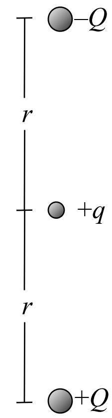
D) Two additional charges equal to $+Q$ are placed the same distance r from $+q$ as shown to the right. Consider the following two statements:

1. "The force on $+q$ is now three times as great, because there are now three other charges a distance r away."
2. "The force on $+q$ is the same as it was before, because the force from the new charge on the right cancels the force from the new charge on the left."

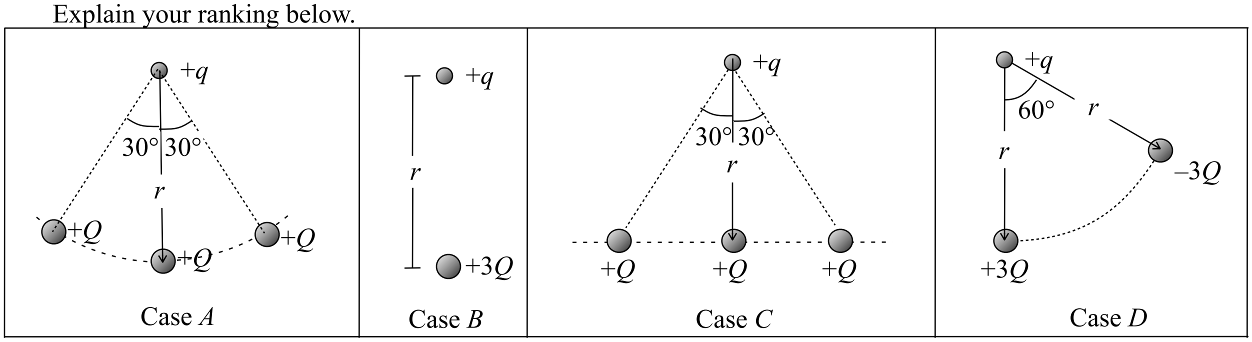
Which, if either, of these statements is correct? Explain.



E) The two additional charges are removed and a negative charge $-Q$ is placed above $+q$ at an equal distance r as shown to the right. Consider the following statement: "By Coulomb's law, the force from the top charge is negative, and the force from the bottom charge is positive, so there is no net force on $+q$." What's wrong with this statement?

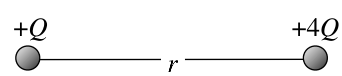


F) Rank the four cases in the diagram according to the *magnitude* of the net force on the $+q$ charge. Explain your ranking below.



3. Two free point charges $+Q$ and $+4Q$ are a distance r apart. A third charge q is placed between them at a distance x from $+Q$ so that it feels no net force.

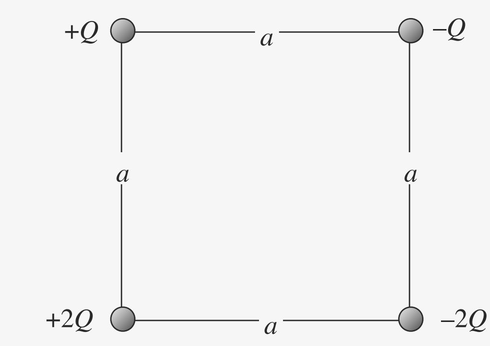
A) What is the sign of the third charge? Explain.



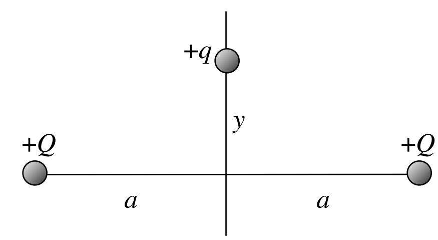
B) Use proportional reasoning to find x in terms of r. Explain.

C) Determine the magnitude and sign of q in terms of Q so that *all three charges* are at equilibrium.

4. Consider the four point charges shown to the right at the corners of a square of edge a.
   A) Draw three vectors at each corner representing the magnitude and direction of the electric force on that point charge due to the other three. Label each vector by its magnitude in units of $\frac{kQ^2}{a^2}$ (eg. 1, 2, etc.)
   B) The 12 vectors you drew form 6 Third Law pairs. Identify these by the letters A through F.
   C) If the upper-right charge were to be made positive, which of the 12 vectors would change, and how would they change?

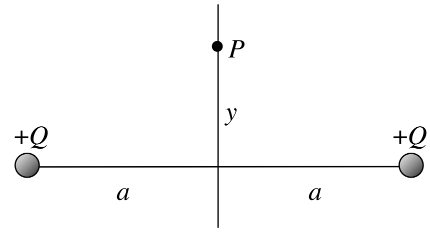


5. Two equal point charges $+Q$ are placed on the x-axis at a distance a from the origin, as shown to the right. A third charge $+q$ is placed on the y-axis at a distance y from the origin.
   A) On the diagram, draw the forces on $+q$ due to each of the other two charges.
   B) What is the direction of the net force on $+q$ due to the other two charges?



   C) Find the magnitude of the net force on $+q$ due to the other charges.
   D) Does your expression make sense when y = 0? Explain.
   E) Find the value of y such that the force on $+q$ is a maximum.
   F) The magnitude of the third charge is now doubled to $+2q$.
      i) How does the net force on $+2q$ compare to the force on $+q$?
      ii) How, if at all, is the value of y found in part E) changed? Explain.

The Electric Field



The diagram to the right shows the situation on the previous page, with the charge $+q$ removed, and its place marked by point P. Your answers to part F) should indicate to you that the *electric force on a charge placed at point P is directly proportional to the charge placed there*. Once you have done the necessary calculations to find the force on a given charge at P, it is not necessary to repeat these calculations to find the force on any other charge at P.

Write the expression in italics above as an equation, using the letter E for the proportionality constant:

The proportionality constant is called the *electric field* at P. What are the units of electric field? What is the magnitude of the electric field at point P? Does this quantity have a maximum at the value of y found in part E) on the previous page? Explain.

If we were to place a negative charge $-q$ at point P, what would be the *direction* of the force on this charge?

Would the *magnitude* of the force on this charge be the same as the magnitude of the force on an equal but positive charge? Explain.

The electric field is a *vector field*, with a magnitude and direction at every point in space. The direction of the electric field is defined (arbitrarily) as the direction of the force that *would* occur if a *positive* charge were placed at the point in question. We therefore have the following recipe for determining the electric field vector at any point:

> Place a small positive test charge, say q_o, at the point. Measure the electrical force $\vec{F}$ on the charge, and divide the force by q_o. The result is the electric field $\vec{E}$, which points in the direction of the force on the positive test charge, and has a magnitude F/q_o.

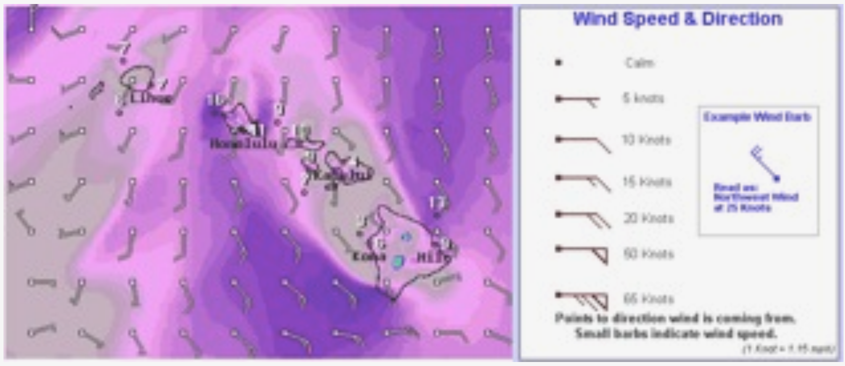
In a similar fashion, we can define the *gravitational field* as a vector field in space, symbolized by $\vec{g}$. Write a recipe for determining the gravitational field similar to the one above:

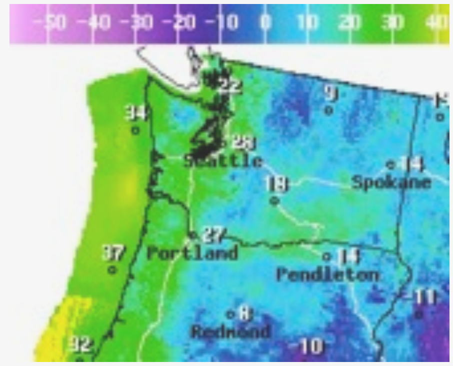
Place a small...

What is $\vec{g}$ at the surface of the earth, in magnitude and direction, with units?

Fields

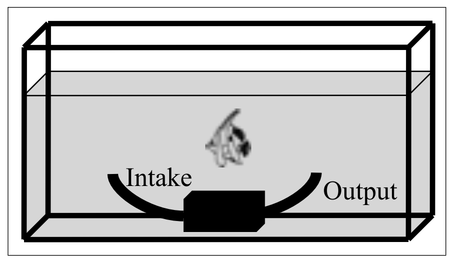
The gravitational and electric fields are both *vector fields*; each is a set of points together with a vector defined at each point. As another example, consider a set of points on the surface of the earth, together with the wind velocity vectors at each point. Such a vector field is depicted in the diagram below on the left, from the National Weather Service web site. Symbols called *wind barbs* are used to show both the direction of the wind and the relative speed.





We can also define a *scalar field* as a set of points together with a scalar defined at each point. The diagram to the right above shows the surface temperature in the Pacific Northwest; this is a scalar field because the value at each point, the temperature, is a scalar. We will encounter both vector and scalar fields in our study of electricity and magnetism.

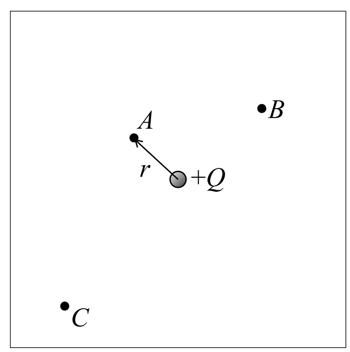
Consider the aquarium tank in the diagram to the right, with a filtering pump at the bottom. There is a velocity vector field for each point in the tank. On the diagram, draw a few vectors to show the general pattern of velocity vectors for this field. How would you describe the field vectors in the neighborhood of the intake?



How would you describe the field vectors in the neighborhood of the output?

In general, points where the field vectors *converge* is called a *sink* for the field, and points where the field vectors *diverge* is called a *source* for the field.

$\vec{E}$ of a Point Charge



Consider an isolated positive point charge $+Q$ in the diagram to the right. Point A is a distance r away from the charge, point B is $2r$ away, and point C is $3r$ away. On the diagram, draw vectors at these three points to show both the directions and the relative magnitudes of the electric field E at these points. How does the length of E_B compare to the length of E_A?

How does the length of E_C compare to the length of E_A?

Use the definition of electric field to write an expression for the magnitudes of the electric field at these points: $E_A =$ $E_B =$ $E_C =$

How would the *directions* of E_A, E_B and E_C differ if the point charge were *negative*?

How would the *magnitudes* of E_A, E_B and E_C differ if the point charge were *negative*?

Use the terms *sink* and *source* to describe the electric field created by point charges.

Use the terms *sink* and *source* to describe the gravitational field created by point masses.

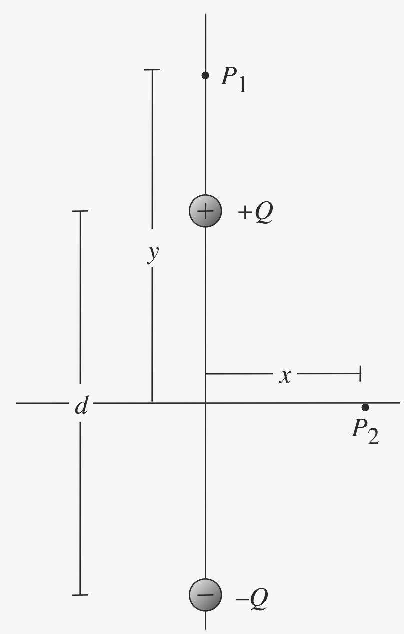
The Electric Dipole

Often in nature there are distributions of electric charge that can be approximated as two equal but opposite charges Q separated by a certain distance d. This configuration is called the *electric dipole*. Consider the electric dipole shown in the diagram below.

A) On the diagram, draw vectors showing the electric field vectors at point P_1, a distance y from the center of the dipole along its axis, due to each of the dipole charges.

B) Show that the magnitude of the electric field at point P_1 is given by

$$E = \frac{2kQyd}{\left(y^2 - \frac{d^2}{4}\right)^2}$$



C) At great distances from the dipole ($y \gg d$), how does E vary with y?

D) On the diagram, draw vectors showing the electric field vectors at point P_2, a distance x from the center of the dipole along a perpendicular bisector, due to each of the dipole charges.

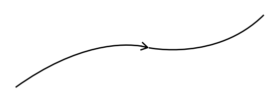
E) Show that the magnitude of the electric field at point P_2 is $E = \dfrac{kQd}{\left(x^2 + \dfrac{d^2}{4}\right)^{\frac{3}{2}}}$

F) At great distances from the dipole ($x \gg d$), how does E vary with x?

G) Expressions concerning electric dipoles are often written in terms of the *dipole moment*, a vector pointing in the direction from the negative to the positive charge, whose magnitude $p = Qd$. Rewrite the expressions from parts C) and F) above in terms of the magnitude of the dipole moment vector $\vec{p}$.

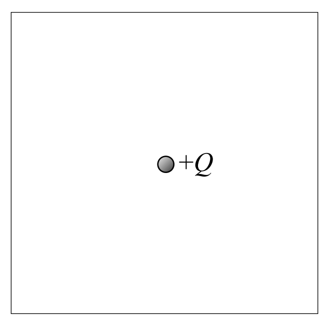
Field Lines

The wind barbs on page 9.5 show one way to represent a vector field. Another way, introduced by Michael Faraday, is by *field lines* which are lines drawn so that the field vectors are always tangent to them. Small arrowheads along the lines indicate the general direction of the field vectors. Field lines are drawn in such a way that the strength of the field (the magnitude of the field vector) is proportional to the density of the field lines. A surprising amount of information can be represented this way.



Field line

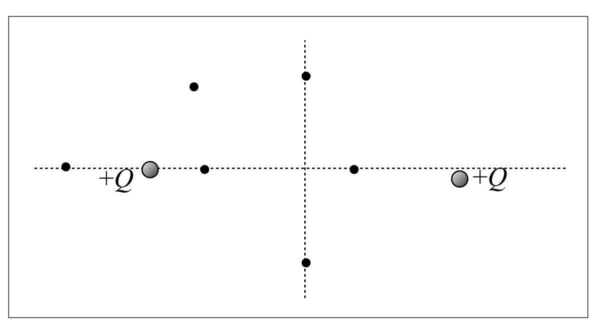
Draw field lines on the diagram to the right to represent the electric field near a positive point charge. Note that the field lines are closer together (more dense) near the charge where the field is strong, and farther apart (less dense) where the field is weaker.



In general, where do field lines *start*?

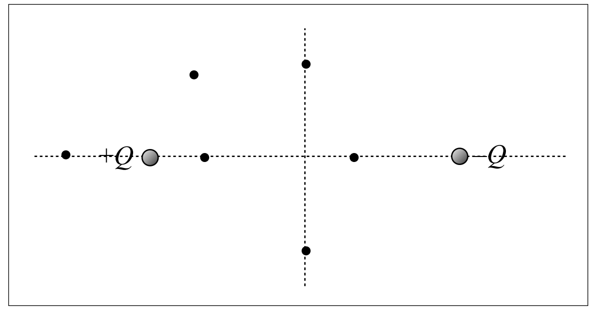
Where do field lines *end*?

The diagram to the right represents a pair of identical positive charges a certain distance apart. Sketch vectors at the points on the diagram to indicate the direction of the electric field at those points. Then sketch field lines throughout the box that show the general pattern of the field vectors. What is the magnitude of the electric field at the midpoint between the charges?



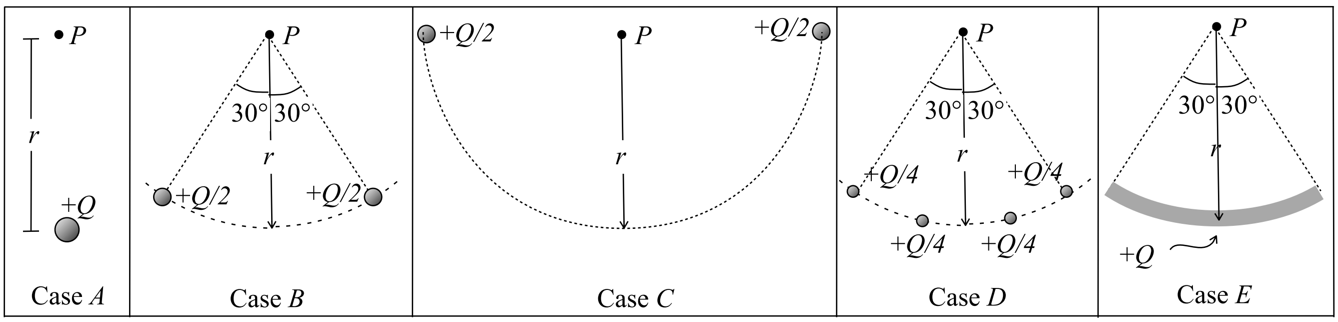
Is the midpoint between the charges a *source*, a *sink*, or *neither*? Explain.

How would the field lines be different if the two point charges were negative?



The diagram to the right represents an electric dipole. Sketch vectors at the points on the diagram to indicate the direction of the electric field at those points. Then sketch field lines throughout the box that show the general pattern of the field vectors.

The diagram below shows 5 situations with different distributions of charge. In case E the charge $+Q$ is spread uniformly over an insulating rod.

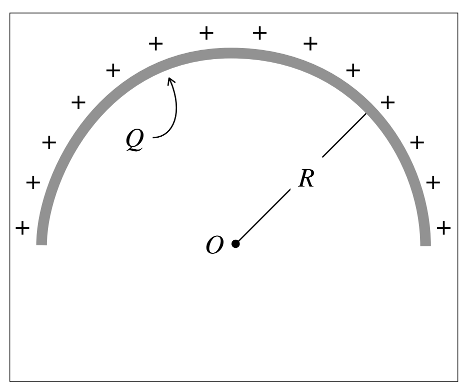


Rank the five cases according to the magnitude of the electric field at point P, from greatest to least. If the electric field has an equal magnitude in any cases , state so explicitly. Explain your ranking.

Continuous Charge Distributions

Case E above is an example of a *continuous charge distribution*, where we consider the charge to be spread through a length, area or volume. We divide the distribution into small pieces, each of which is considered a point charge with a differential amount of charge dq. Each dq contributes its share $d\vec{E}$ to the total electric field at a given point.

Examples



1. A thin plastic rod is bent into a semicircle of radius R, and charged uniformly with a total charge $+Q$.
   A) On the diagram, identify an arbitrary differential section of the rod as having a charge dq. Draw a vector indicating $d\vec{E}$, the differential contribution to the electric field at O due to dq.
   B) Express the magnitude dE in terms of dq and R:
   C) Choose parameters to uniquely identify the location and size of dq and show these parameters on the diagram.

D) What is the direction of the electric field at O due to the rod? Explain.

E) On the diagram, show the components of the vector $d\vec{E}$ that are parallel and perpendicular to the direction of $\vec{E}$. As you integrate over all dq's that make up the rod, which components will add and which will cancel? Explain.

F) Although $\vec{E} = \int d\vec{E}$ is true as a vector equation, $E = \int dE$ is *not* true as a scalar equation. Explain the difference.

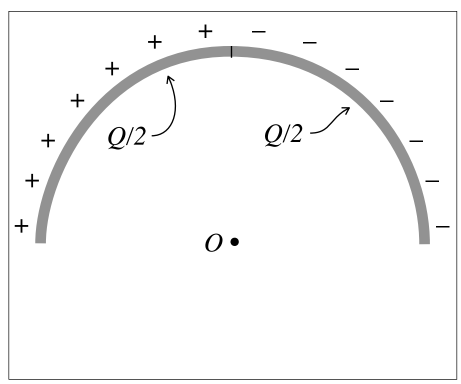
G) Write the scalar integral equation for E that *is* true, according to your chosen parameters.

H) Use your chosen parameters to complete the following ratio: $\dfrac{dq}{Q} =$

I) Choose appropriate limits of integration and find the value of $\vec{E}$ at O in terms of k, Q, and R.

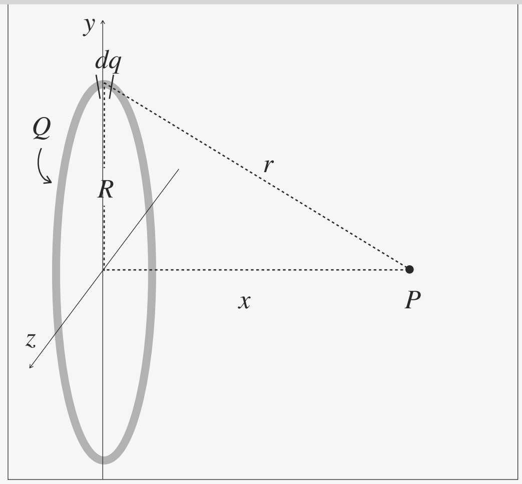
J) The charge on the right side of the rod is now replaced with an equal amount of negative charge. On the diagram below, draw vectors $\vec{E}_{\mathrm{L}}$ and $\vec{E}_{\mathrm{R}}$ representing the electric field at O due to the left and right sides of the rod, respectively. Draw the vector $\vec{E}$ representing the new field at O.

K) Determine the magnitudes of $\vec{E}_{\mathrm{L}}$ and $\vec{E}_{\mathrm{R}}$, and explain your reasoning.



L) Determine the magnitude of $\vec{E}$.

2. The figure to the right shows a thin ring of radius R in the y-z plane with a total positive charge Q uniformly distributed around the ring. At the top of the ring is a representative section of the ring of charge dq. Point P is on the x-axis at a distance x from the center of the ring.



A) On the diagram, draw a vector representing the differential contribution $d\vec{E}$ to the electric field at P due to dq. Express the magnitude of $d\vec{E}$ using the parameters given: $dE =$

B) What is the direction of the electric field at P due to the ring? Explain your reasoning.

C) On the diagram, show the components of the vector $d\vec{E}$ that are parallel and perpendicular to the direction of $\vec{E}$. Express these components *without using trigonometric functions*. As you integrate over all dq's that make up the ring, which components will add and which will cancel? Explain.

D) Write the scalar integral equation for E using the given parameters.

E) Move all constants out of the integral and perform the integration, expressing the result in terms of R, x, and the total charge Q.

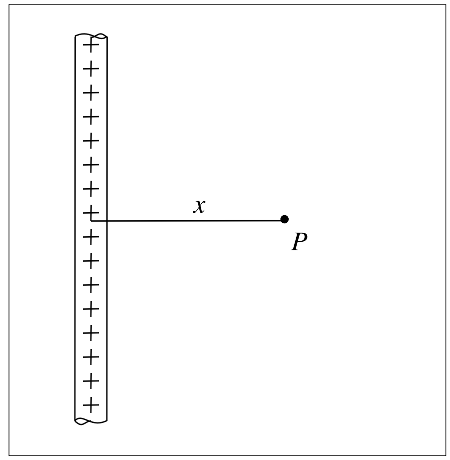
F) Does your expression make sense when $x = 0$? Explain.

G) Does your expression make sense when $x \gg R$? Explain.

H) Find the location (value of x) for which the electric field is a maximum.

I) We often describe the charge on a continuous distribution in terms of the *linear charge density* λ (lambda), which is the charge per unit length. What is the linear charge density of the ring? $\lambda =$

3. Consider an infinitely long line of charge, with charge density λ, as shown to the right. Point P is a distance x from the line.



A) Choose a representative dq on the line and identify its size and location with appropriate parameters.

B) Draw a vector $d\vec{E}$ at P representing the differential contribution to the electric field due to dq. Express the magnitude of $d\vec{E}$ using the parameters chosen: $dE =$

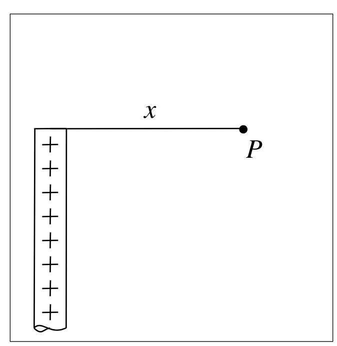
C) What is the direction of the electric field at P due to the infinite line? Explain your reasoning.

D) On the diagram, show the components of the vector $d\vec{E}$ that are parallel and perpendicular to the direction of $\vec{E}$. Express these components in terms of appropriate trigonometric functions. As you integrate over all dq's that make up the infinite line, which components will add and which will cancel? Explain.

E) Move all constants out of the integral and express all remaining variables in terms of trigonometric functions.

F) Choose appropriate limits and perform the integration.

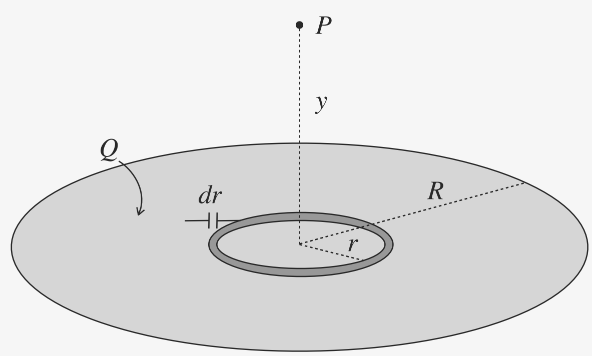
G) The infinite line is now cut in half at the point nearest point P as shown. On the diagram, indicate the general direction of the electric field $\vec{E}_P$ at P. Determine the x-component of $\vec{E}_P$, either by modifying your solution above, or by using symmetry.



H) Determine the y-component of $\vec{E}_P$.

G) What is the exact direction of $\vec{E}_P$?

4. The diagram to the right shows a uniformly charged disk of radius R and positive charge Q. Point P is a distance y from the center of the disk, along its axis. A differentially thin ring of radius r and charge dq is identified.



A) What is the direction of $\vec{E}_P$?

B) Draw a vector $d\vec{E}$ at P representing the differential contribution to the electric field due to dq. By suitably modifying the expression from example 2 E), page 9.10, express the magnitude of $d\vec{E}$ using the parameters given:

$dE =$

C) Complete the following ratio and simplify: $\dfrac{dq}{Q} =$

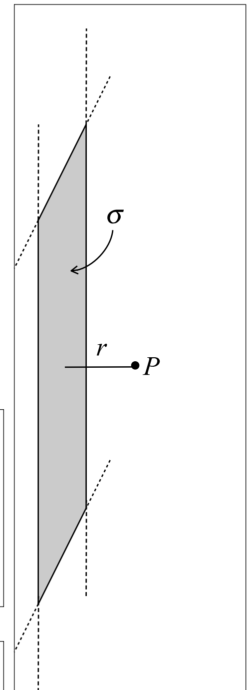
D) Integrate with appropriate limits to find the magnitude of $\vec{E}_P$.

E) We often describe the charge on a surface like this in terms of the *surface charge density* σ (sigma), which is the charge per unit area. What is the surface charge density of the disk? $\sigma =$

F) Express the magnitude of $\vec{E}_P$ in terms of σ: $E_P =$

G) What does this expression reduce to at points very close to the disk ($y \ll R$)? $E_P =$

5. The diagram to the right shows a uniformly charged *infinite sheet* with positive surface charge density σ. Point P is a distance r from the sheet.



A) On the diagram, draw a vector at P showing the electric field $\vec{E}_P$.

B) Based on your answer to example 4 G) on the previous page, what is the magnitude of $\vec{E}_P$?

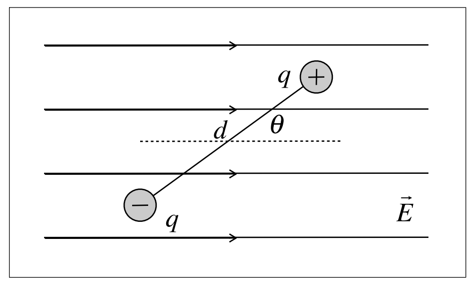
$E_P =$

C) On the diagram, draw a series of field lines on the right side of the sheet.

D) What is the magnitude of the electric field at another point a distance $2r$ from the sheet? Explain how your answer is consistent with the expression in part B) and the field lines you drew in part C).

E) Describe the field lines on the left side of the sheet.

6. A uniform electric field is one which is constant in magnitude and direction. The diagram to the right shows a uniform field $\vec{E}$ pointing in the positive x-direction in a region of space. An electric dipole is placed in the existing field at an angle θ to the field as shown to the right.



A) On the diagram, draw vectors indicating the forces acting on the two charges due to the uniform electric field.

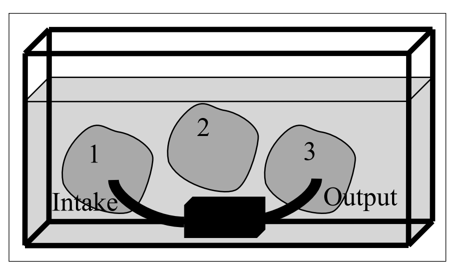
B) What is the magnitude of the force on each charge? $F =$

C) What is the net force on the dipole? $\Sigma F =$

C) What is the magnitude of the net torque on the dipole about its center? $\Sigma\tau =$

D) Recall that the electric dipole moment $\vec{p}$ is a vector pointing from the negative to the positive charge, with magnitude qd. Write an expression relating the torque, the dipole moment and the electric field using a vector cross-product. $\Sigma\vec{\tau} =$

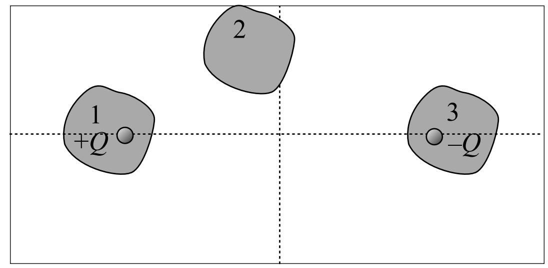
Consider again the aquarium tank from page 9.5. Three mesh bags are submerged in the tank, one enclosing the intake, one the output, and a third enclosing neither. The tiny holes in the mesh allow water to flow through freely. Imagine that you could keep track of all the water flowing either way through these holes.



A) Make a statement about the total flow of water into and out of each bag.

B) If a fourth bag enclosed both the intake and output, what would you say about the total flow of water for this bag?

C) Now consider an electric dipole, such as the one on page 9.8. Imagine three bags as in the aquarium, and you can keep track of the electric field lines passing either way through the surface of each bag. Make a statement comparing what you would find for the three bags.

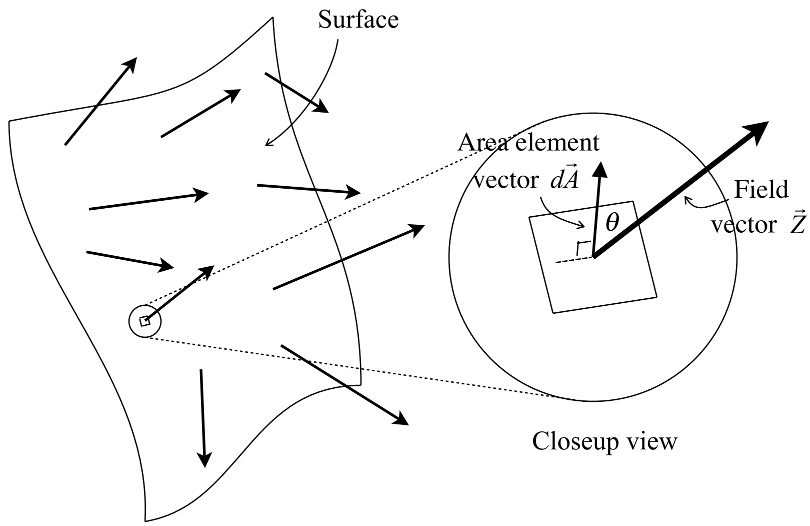


The above discussion is the essence of *Gauss' Law*, which relates the electric field lines passing through a closed surface (such as a bag) to the net electric charge inside the surface. First we need to have a precise way of keeping track of the field lines passing through the surface.

Flux of a Vector Field

Consider an arbitrarily shaped surface in a region of space where some vector field $\vec{Z}$ exists. We are interested in keeping track of how much of the field goes through the surface, so we need to know the component of $\vec{Z}$ that is perpendicular to the surface.

We divide the surface up into differentially small patches, or *area elements*, which are so small that they are flat (even if the surface is curved), and there is essentially one field vector located at that area element. We represent the area element as a vector $d\vec{A}$ whose length is proportional to the (differentially small) area, and whose direction is perpendicular to the surface.

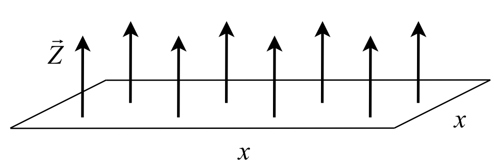


We define the *flux* of the vector field over this surface as follows: For each area element, take the dot product of $d\vec{A}$ with the corresponding field vector. Add these dot products for each area element that covers the surface. The resulting scalar quantity is the flux, symbolized by Φ (phi). Since it takes an infinite number of $d\vec{A}$'s to cover the surface, this sum is really an integral: $\Phi =$

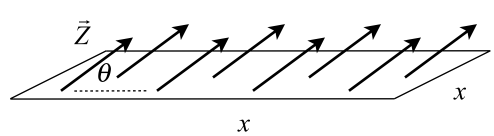
The sign of the flux in this case is arbitrary, because there are two different directions that $d\vec{A}$ can have and still be perpendicular to the surface. We are usually interested in the flux a *closed* surface, in which case the $d\vec{A}$ vectors by definition always point *outward* from the surface. If the surface over which the flux is calculated is closed, then a special symbol is used for the integral: $\oint$

Examples

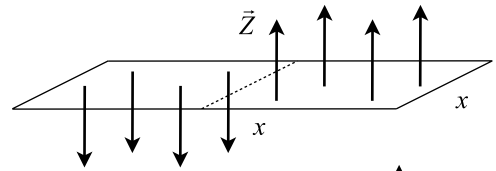
1. A square surface with edge x is held perpendicular to a uniform field $\vec{Z}$ as shown. What is the flux of $\vec{Z}$ for this surface?



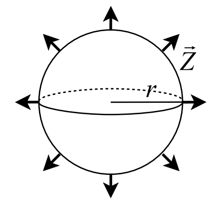
2. The same surface is now held so that the uniform $\vec{Z}$ field is at an angle θ to the surface. What is the flux of $\vec{Z}$ for this surface?



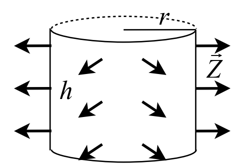
3. The same surface is now held as in example 1, but the field on the left half points down. What is the flux of $\vec{Z}$ for this surface?



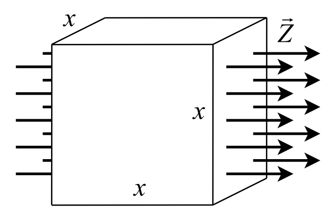
4. At every point on the surface of a sphere of radius r, a field $\vec{Z}$ has the same magnitude and points radially outwards as shown to the right. What is the flux of $\vec{Z}$ for this surface?



5. At every point on the sides of a cylinder of radius r and height h, a field $\vec{Z}$ has the same magnitude and points radially outwards as shown to the right. What is the flux of $\vec{Z}$ for this surface?



6. A cube of side x is placed in a uniform horizontal field $\vec{Z}$ so that the left and right sides of the cube are perpendicular to $\vec{Z}$ as shown to the right.



A) What is the flux of $\vec{Z}$ for the right side of the cube?

B) What is the flux of $\vec{Z}$ for the left side of the cube?

C) What is the flux of $\vec{Z}$ for the front side of the cube?

D) What is the flux of $\vec{Z}$ for the entire cube?

Gauss' Law

We can now state Gauss' Law:

> *The flux of the electric field over a closed surface is directly proportional to the net charge inside the surface.*

We use the symbol q_{in} to represent the net charge inside the closed surface, so we can state Gauss' Law simply as:

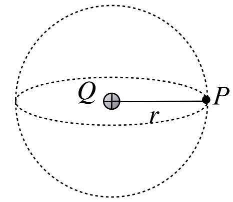
$$\Phi_E \propto q_{\text{in}} .$$

The proportionality constant is written as ε_o (epsilon naught). For historical reasons, it is written on the left side of the equation, so:

$$\begin{aligned} \varepsilon_o \Phi_E &= q_{\text{in}} \\ \varepsilon_o \oint \vec{E} \bullet d\vec{A} &= q_{\text{in}} \end{aligned} \quad \text{where } \varepsilon_o = \frac{1}{4\pi k}, \ k = 9\times10^9 \text{ Nm}^2/\text{C}^2$$

Gauss' Law can be used to calculate the electric field due to continuous charge distributions that have certain kinds of symmetry: spherical, cylindrical, and plane. These distinctions will become clear as we consider examples.

Example



Consider an isolated positive point charge Q. We wish to find the electric field at point P, a distance r from Q. Point P represents *any* point at the same distance r, since the field of a point charge is *spherically symmetric*. We choose a closed surface (called a *Gaussian surface*) in the shape of a sphere of radius r, centered on Q as shown to the right. The Gaussian surface thus passes through P and follows the spherical symmetry of the field.

A) Write Gauss' Law:

B) What is q_{in} in this case?

C) What is the flux of $\vec{E}$ in this case?

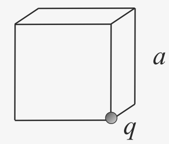
D) Solve your expression for E: $E =$ Compare with your expressions on page 9.5.

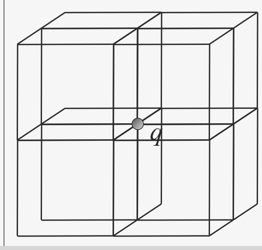
Discussion Questions

1. A point charge is placed at the center of a spherical Gaussian surface. Under the following conditions, describe how (if at all) the flux of the electric field over the surface is *changed*? Explain your reasoning in each case.
   A) the surface is replaced by a cube of the same volume
   B) the surface is replaced by a sphere of one-half the volume
   C) the charge is moved off-center in the original sphere, remaining inside
   D) a second charge is placed near, and outside the original sphere
2. A Gaussian surface completely encloses an electric dipole, and no other charge. What can you say about Φ_E for this surface?
3. In Gauss' Law, $\varepsilon_o \oint \vec{E} \bullet d\vec{A} = q_{\text{in}}$, is $\vec{E}$ necessarily the electric field due to q_{in} alone? Explain.
4. Suppose that a Gaussian surface encloses no net charge. Does Gauss' Law require that $\vec{E}$ be zero for all points on the surface? Explain.

5. Conversely, if $\vec{E}$ is zero everywhere on the Gaussian surface, does Gauss' Law require that there be no net charge inside? Explain.

6. A positive point charge q is placed at the corner of a cube of edge a. The cube has six faces, three of which are in contact with q and three of which are not. Find the flux of the electric field over one of each group of faces. Use symmetry arguments rather than a direct integration. Explain your reasoning. (Hint: consider the larger cube of edge $2a$, with q at its center.)





7. **Gauss' Law for Gravity** Recall that Gauss' Law for the electric field states that the flux of the electric field over a closed surface is proportional to the net electric charge inside. A more general form of Gauss' Law would state that the flux of *any* vector field over a closed surface is proportional to the net "source" enclosed by the surface.
   A) If we write Gauss' Law for the electric field as $\Phi_E \propto q_{\text{in}}$, how would we write Gauss' Law for the gravitational field? $\Phi_g \propto$
   B) Recall the expression for the gravitational field strength at a distance r from a spherically symmetric mass M. Then use this to determine the appropriate proportionality constant, and write Gauss' Law for gravity (be careful with the sign of the flux).

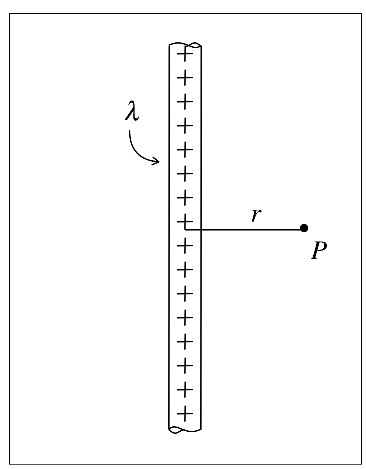
Applying Gauss' Law

In general, applying Gauss' Law involves these steps:

1. Choose a Gaussian surface that reflects the symmetry of the field (spherical, cylindrical or plane), and passes through the point at which you want to find $\vec{E}$.
2. Write Gauss' Law and use symmetry arguments to simplify the integral.
3. Evaluate the integral and solve for the field.

Examples

1. Consider an infinite line of positive charge, charge density λ. You wish to find the electric field E_P at point P, a distance r from the line.
   A) What kind of symmetry does the field have?
   B) On the diagram to the right, draw a Gaussian surface that reflects this symmetry and passes through P. Note that although the line is infinite, your Gaussian surface will be finite. Indicate parameters that specify the size and shape of the Gaussian surface.
   C) What is q_{in} for your Gaussian surface?



Your Gaussian surface should have a top, a bottom, and a curved side. The integral in Gauss' Law therefore becomes:

$$\oint \vec{E} \bullet d\vec{A} = \int_{\text{top}} \vec{E} \bullet d\vec{A} + \int_{\text{bottom}} \vec{E} \bullet d\vec{A} + \int_{\text{side}} \vec{E} \bullet d\vec{A}$$

D) Why don't the three integral symbols on the right have circles on them?

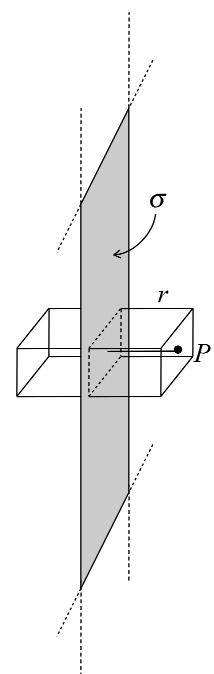
E) Of the three integrals, which, if any, evaluate to zero? Explain why. (Hint: Where do the $d\vec{A}$ vectors point?)

F) For which of the remaining integrals is the dot product $\vec{E} \bullet d\vec{A}$ equal to the scalar product $E\,dA$? Explain why.

G) Use symmetry to determine the total flux for your Gaussian surface. Be explicit about how you are using symmetry.

H) Solve for E_P and compare with the field you found on page 9.11.

2. Consider an infinite sheet of positive charge, charge density σ. You wish to find the electric field E_P at point *P*, a distance *r* from the sheet. A Gaussian surface (a rectangular box) has been drawn on the diagram. Point *P* lies on one face of the box, and the box extends an equal distance behind the sheet.



A) What kind of symmetry does the field have?

B) Label the remaining parts of the box so that you can determine its exact size. What is q_{in} for this Gaussian surface?

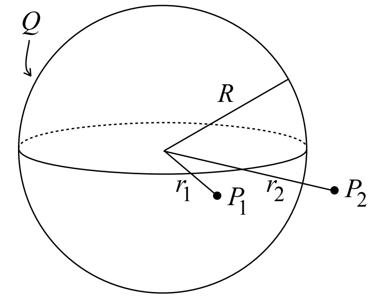
C) The flux for this surface is an integral over the six sides of the box. Which of these six integrals evaluates to zero? Explain why.

E) For which of the remaining integrals is the dot product $\vec{E} \bullet d\vec{A}$ equal to the scalar product $E\,dA$? Explain why.

F) Use symmetry to determine the total flux for your Gaussian surface. Be explicit about how you are using symmetry.

G) Solve for E_P and compare with the field you found on page 9.13.

3. Consider a thin spherical shell of radius R with a total positive charge $+Q$ uniformly over its surface. Point P_1 is inside the sphere at a distance r_1 from the center, and P_2 is outside the sphere at a distance r_2 from the center. Use Gauss' Law to find the magnitude of the electric field at these points. In each case, describe the kind of symmetry, specify the Gaussian surface, and use symmetry to evaluate the integral.



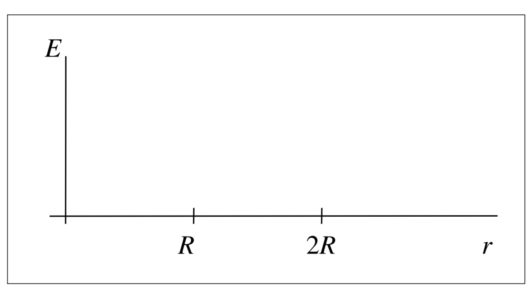
A) Find the magnitude of the field at P_1

B) Find the magnitude of the field at P_2

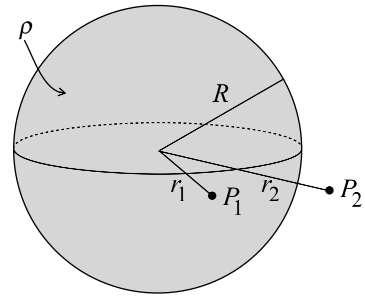
C) On the axes to the right, graph the magnitude of the electric field as a function of the distance from the center of the sphere, indicating the values of E at $r = R$ and $r = 2R$.

D) What is the surface charge density σ of the shell?

E) Express the electric field at the surface of the sphere in terms of σ



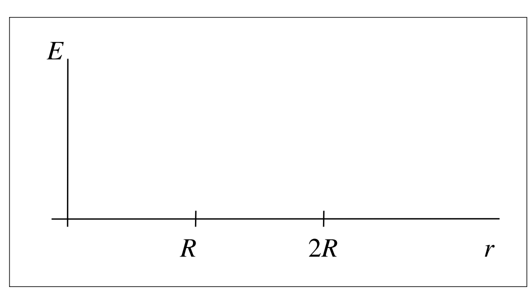
4. Consider a solid sphere of radius R with a uniform positive volume charge density (charge per unit volume) ρ (rho). Point P_1 is inside the sphere at a distance r_1 from the center, and P_2 is outside the sphere at a distance r_2 from the center. Use Gauss' Law to find the magnitude of the electric field at these points. In each case, describe the kind of symmetry, specify the Gaussian surface, and use symmetry to evaluate the integral.



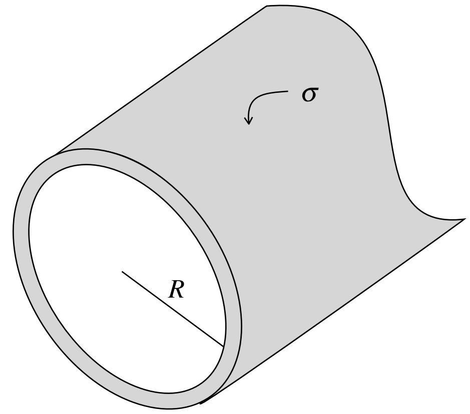
A) Find the magnitude of the field at P_1

B) Find the magnitude of the field at P_2

C) On the axes to the right, graph the magnitude of the electric field as a function of the distance from the center of the sphere, indicating the values of E at $r = R$ and $r = 2R$.



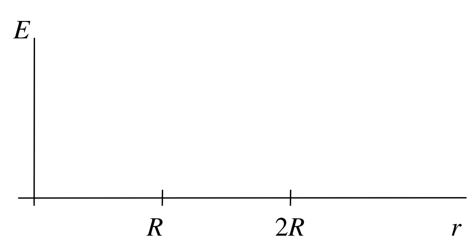
5. Consider a long, thin cylindrical shell of radius R with a positive surface charge density σ uniformly over its surface. Use Gauss' Law to find the magnitude of the electric field both inside ($r < R$) and outside ($r > R$) the shell. In each case, describe the kind of symmetry, specify the Gaussian surface, and use symmetry to evaluate the integral.



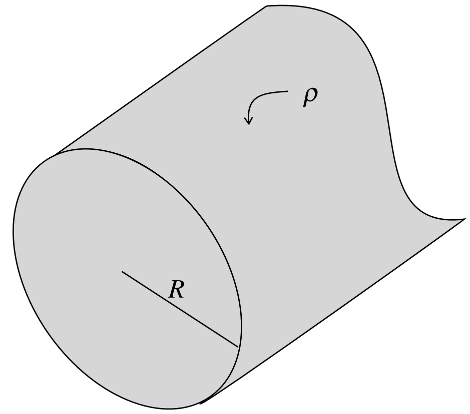
A) ($r < R$)

B) ($r > R$)

C) On the axes to the right, graph the magnitude of the electric field as a function of the distance from the center of the shell, indicating the values of E at $r = R$ and $r = 2R$.



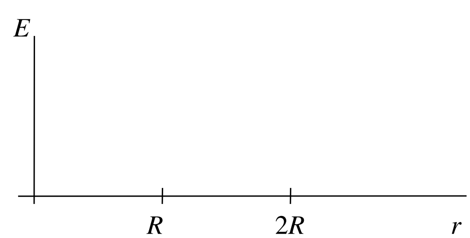
6. Consider a long, solid cylinder of radius R with a positive volume charge density ρ uniformly throughout its volume. Use Gauss' Law to find the magnitude of the electric field both inside ($r < R$) and outside ($r > R$) the shell. In each case, describe the kind of symmetry, specify the Gaussian surface, and use symmetry to evaluate the integral.



A) ($r < R$)

B) ($r > R$)

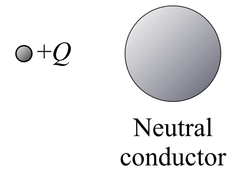
C) On the axes to the right, graph the magnitude of the electric field as a function of the distance from the center of the shell, indicating the values of E at $r = R$ and $r = 2R$.



Conductors and Insulators

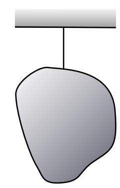
Some materials, called *conductors*, have the property that there are free charges that move very easily in response to an electric field. Most metals are conductors. Other materials, called *insulators*, do not allow charge to flow through them. Wood, plastic, and glass are insulators.

Consider a positive charge $+Q$ brought near a conductor with no net charge. Describe what happens to the free charge on the conductor.



Is there a net electric force between the charge and the conductor? Explain.

A piece of conducting material is tied to an insulating thread, and is given a net positive charge. The excess charge rapidly comes to equilibrium. Consider a point within the material. Could there be a nonzero electric field there? Explain.



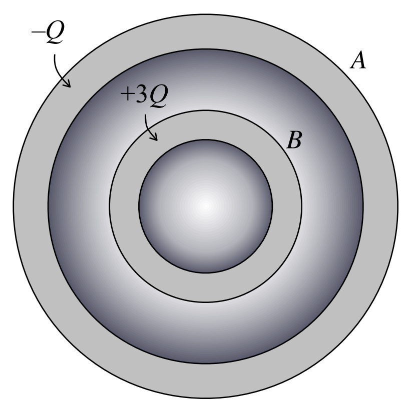
Choose a Gaussian surface that lies completely within the conducting material. What is the value of the electric field at every point on this surface? Explain.

What is the flux of the electric field over this surface? According to Gauss' Law, what is q_{in} for this surface? We have not specified anything about the surface other than the fact that it is completely within the conducting material. Any excess charge must lie outside this surface, yet the surface can be chosen arbitrarily close to the outside of the material. Complete the following statement:

Any excess charge on a solid conductor at equilibrium must lie

Now consider two thick concentric spherical conducting shells *A* and *B* shown in cross-section below, carrying charges $-Q$ and $+3Q$ respectively. Find the charges on the inner and outer surfaces of both shells, and *explain your reasoning using Gauss' Law* where appropriate.

A) Inner surface of *B*:



B) Outer surface of *B*:

C) Inner surface of *A*:

D) Outer surface of *A*:

AP Physics C
Unit 9 Practice Test

Name ___________________________________

Part I
Multiple Choice

1. The electric field $\vec{E}$ just outside the surface of a charged conductor is

   A) directed perpendicular to the surface
   B) directed parallel to the surface
   C) independent of the surface charge density σ
   D) zero
   E) infinite

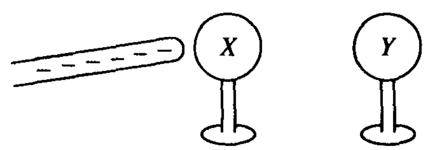
2. A point charge is placed at the center of an uncharged, spherical, conducting shell of radius *R*. The electric fields inside and outside the sphere are measured. The point charge is then moved off center a distance *R*/2 and the fields are measured again. What is the effect on the fields?

   A) Changed neither outside nor inside
   B) Changed inside but not changed outside
   C) Not changed inside but changed outside
   D) Changed inside and outside

3. The net electric flux through a closed surface is

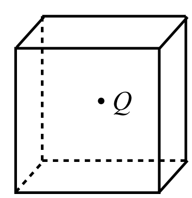
   A) infinite only if there are no charges enclosed by the surface
   B) infinite only if the net charge enclosed by the surface is zero
   C) zero if only negative charges are enclosed by the surface
   D) zero if only positive charges are enclosed by the surface
   E) zero if the net charge enclosed by the surface is zero

4. Two metal spheres that are initially uncharged are mounted on insulating stands, as shown to the right. A negatively charged rubber rod is brought close to, but does not make contact with, sphere *X*. Sphere *Y* is then brought close to *X* on the side opposite to the rubber rod. *Y* is allowed to touch *X* and then is removed some distance away. The rubber rod is then moved far away from *X* and *Y*. What are the final charges on the spheres?



	Sphere *X*	Sphere *Y*
A)	Zero	Zero
B)	Negative	Negative
C)	Negative	Positive
D)	Positive	Negative
E)	Positive	Positive

For questions 5 and 6: The point charge *Q* shown to the right is at the center of a cube.



5. If the cube is a Gaussian surface, then the electric flux through any one of the faces of the cube is

   A) $\dfrac{Q}{\varepsilon_o}$ B) $\dfrac{Q}{4\pi\varepsilon_o}$ C) $\dfrac{Q}{3\varepsilon_o}$ D) $\dfrac{Q}{6\varepsilon_o}$ E) $\dfrac{Q}{12\varepsilon_o}$

6. If the cube is a metal box that is isolated, ungrounded, and uncharged, which of the following is true?

   A) The net charge on the outside surface of the box is *Q*.
   B) The electric field inside the box is zero.
   C) The electric field inside the box is uniform, but not zero.
   D) The electric field outside the box is zero everywhere.
   E) The electric field outside the box is the same as if only the point charge (and not the box) were there.

7. A small object has charge Q. A charge q is removed from it and placed on another small object. The two objects are held a certain distance apart. For the repulsive force between the two objects to be a maximum, q should be

   A) $2Q$
   B) Q
   C) $Q/2$
   D) $Q/4$
   E) 0

For questions 8 and 9: A positively charged insulating rod is brought close to an object that is suspended by an insulating thread.

8. If the object is attracted toward the rod we can conclude

   A) that the object is positively charged
   B) that the object is negatively charged
   C) that the object is an insulator
   D) that the object is a conductor
   E) none of the above

9. If the object is repelled away from the rod we can conclude

   A) that the object is positively charged
   B) that the object is negatively charged
   C) that the object is an insulator
   D) that the object is a conductor
   E) none of the above

10. Two point charges have equal and opposite magnitudes $+Q$ and $-Q$. Aside from infinite distances, the electric field is zero

   A) midway between $+Q$ and $-Q$
   B) on the perpendicular bisector of the line joining $+Q$ and $-Q$, but not on the line itself
   C) on the line joining $+Q$ and $-Q$, to the side of Q opposite $-Q$
   D) on the line joining $+Q$ and $-Q$, to the side of $-Q$ opposite Q
   E) at none of these places (there is no point at a finite distance)

11. The electric field due to a charge uniformly distributed over the surface of a spherical shell is zero

   A) everywhere
   B) nowhere
   C) only at the center of the shell
   D) only inside the shell
   E) only outside the shell

12. A charge Q is distributed uniformly throughout an insulating sphere of radius R. The magnitude of the electric field at a point $R/2$ from the center of the sphere is

   A) $\dfrac{Q}{4\pi\varepsilon_o R^2}$

   B) $\dfrac{Q}{\pi\varepsilon_o R^2}$

   C) $\dfrac{3Q}{4\pi\varepsilon_o R^2}$

   D) $\dfrac{Q}{8\pi\varepsilon_o R^2}$

   E) none of these

13. Charge is distributed uniformly along a very long straight wire. The electric field 2 cm from the wire has magnitude 20 N/C. The magnitude of the electric field 4 cm from the wire is

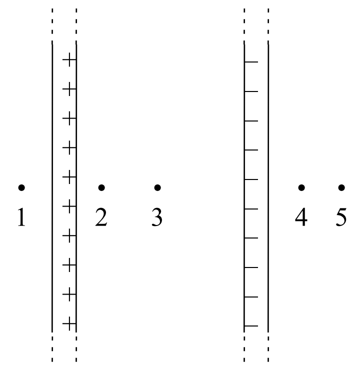
A) 120 N/C
B) 80 N/C
C) 40 N/C
D) 10 N/C
E) 5 N/C

14. A long line of charge with charge per unit length λ_ℓ runs along the axis of a cylindrical conducting shell that carries a charge per unit length λ_c. The charge per unit length on the inner and outer surfaces of the shell are:

A) λ_ℓ and λ_c
B) $-\lambda_\ell$ and $\lambda_c + \lambda_\ell$
C) $-\lambda_\ell$ and $\lambda_c - \lambda_\ell$
D) $\lambda_c + \lambda_\ell$ and $\lambda_c - \lambda_\ell$
E) $\lambda_\ell - \lambda_c$ and $\lambda_c + \lambda_\ell$

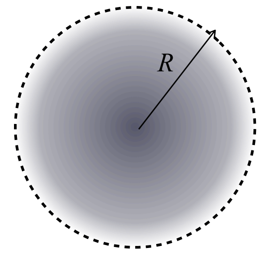
15. Two very large parallel conducting plates carry charge of equal magnitude, distributed uniformly over their inner surfaces as shown to the right. The ranking of the magnitude of the electric field at points 1 - 5 is

A) 1 < 2 < 3 < 4 < 5
B) 5 < 4 < 3 < 2 < 1
C) 1 = 4 = 5 < 2 = 3
D) 2 = 3 < 1 = 4 < 5
E) 2 = 3 < 1 = 4 = 5



1.A 2.B 3.E 4.D 5.D 6.A 7.C 8.E 9.A 10.E 11.D 12.D 13.D 14.B 15.C

Part II
Show your work
Credit depends on the quality and clarity of your explanations



1. A spherical cloud of charge of radius R contains a total charge $+Q$ with a nonuniform volume charge density that varies according to the equation

$$\rho(r) = \rho_o\left(1 - \frac{r}{R}\right) \text{ for } r \le R \text{ and}$$

$$\rho = 0 \text{ for } r > R$$

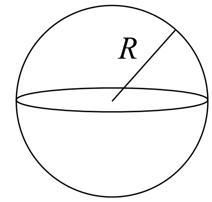
where r is the distance from the center of the cloud. Express all answers in terms of Q, R, and fundamental constants.

A) Determine the magnitude E of the electric field as a function of r for $r > R$.

B) Derive an expression for ρ_o.

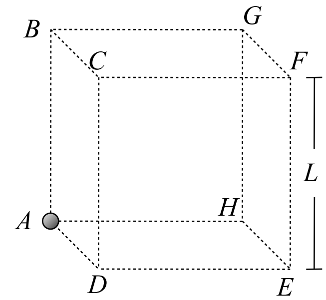
C) Determine the magnitude E of the electric field as a function of r for $r \le R$.

2. A nonconducting, thin, spherical shell of radius *R* has a uniform surface charge density σ on its outside surface and no charge anywhere else inside.



A) Use Gauss's law to prove that the electric field inside the shell is zero everywhere. Draw or describe the Gaussian surface that you use.
B) The charges are now redistributed so that the surface charge density is no longer uniform. Is the electric field still zero everywhere inside the shell? Justify your answer.

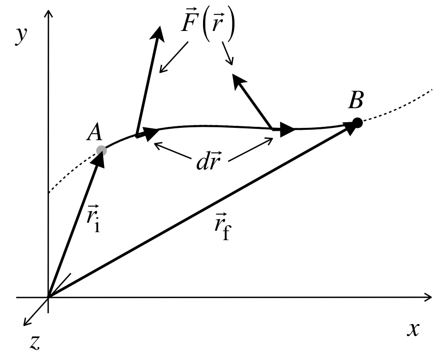
Now consider a small conducting sphere with charge $+Q$ whose center is at corner *A* of a cubical surface, as shown to the right.



C) For which faces of the surface (eg., *ABCD*), if any, is the electric flux through that face equal to zero? Explain your reasoning.
D) At which corner(s) of the surface does the electric field have the least magnitude? Explain.
E) Determine the electric field strength at the position(s) you have indicated in part D) in terms of Q, L, and fundamental constants, as appropriate.
F) Given that one-eighth of the sphere at point *A* is inside the surface, calculate the electric flux through face *CDEF*.

AP Physics C
Unit 10 Name________________________

Review of Work



The diagram to the right represents a particle moving on a path from point *A* (position vector $\vec{r}_i$) to point *B* (position vector $\vec{r}_f$), while a net force $\vec{F}(\vec{r})$ varies both in magnitude and direction along the path. The vector $d\vec{r}$ represents a differential displacement along the path. Write the integral that represents the work done by the net force along this path: $W =$

Describe the relationship between the vectors $\vec{F}(\vec{r})$ and $d\vec{r}$ when the work being done is instantaneously:

A) positive:

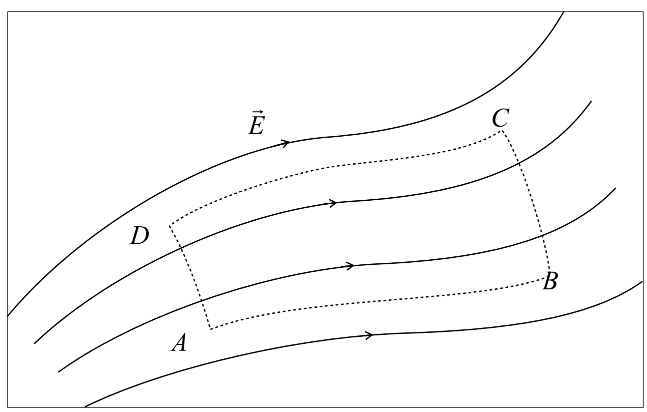
B) negative:

C) zero:

Can the instantaneous work be positive even though the total work for the whole path is negative, or vice versa? Explain.

Under what conditions will the speed of the particle at point *B* be *greater than*, *less than*, or *equal to* its speed at point *A*?

Work Done by the Electric Field



Consider a region of space where an electric field exists, as shown to the right. Paths *AB* and *CD* are along the electric field lines, and paths *BC* and *DA* are perpendicular to the field lines. A small positive charge +*q* is moved along various paths. For each path, indicate whether the work done by the electric field is *positive*, *negative*, or *zero*. On the diagram, draw vectors along each path that illustrate your reasoning.

Path *AB*: Path *BC*:

Path *CD*: Path *DA*:

Along path *AB*, the particle moves in the direction of the electric field. According to the diagram, is the strength of the electric force on the charge *greater near A*, *greater near B*, or *equal along AB*? Explain.

How does the work done by the electric field as the charge is moved from *A* to *B* (W_{AB}) compare to the work done as the charge is moved from *B* to *A* (W_{BA})? Explain.

Electric Potential Energy

Recall from Unit 4 that when the force on an object depends only on its position, there is a potential energy function associated with that force. In the case of the electric field, this energy is called *electric potential energy* U_E. Consider the small positive charge $+q$ at each of the four points *A*, *B*, *C*, and *D* in the diagram on the previous page. Rank the electric potential energy of the charge at each of these points, and explain your reasoning.

As the positive charge is moved along each of the following paths, indicate whether the *change* in electric potential energy ΔU_E is *positive*, *negative*, or *zero*:

Path *AB*: Path *BC*: Path *CD*: Path *DA*:

Compare the work done by the electric field for each path (on the previous page) to the change in electric potential energy for that path. Make a general statement relating the work to the change in potential energy:

In view of this statement and the integral expression for work on the top of the previous page, write an expression for the difference between the electric potential energy of the charge at point *A* and the electric potential energy of the charge at point *B*. The expression should involve the charge *q* and the electric field $\vec{E}$, and should be a definite integral with limits indicating the end points of the path with respect to an arbitrary origin: $\Delta U_{AB} =$

If the meaning is clear, the limits of integration are often written as simply *A* and *B*. This integral is called a *path integral*, evaluated for some path connecting the end points. Since the electric force is a *conservative* force, this path integral is independent of the particular path of integration, and depends only on the end points.

Electric Potential Difference

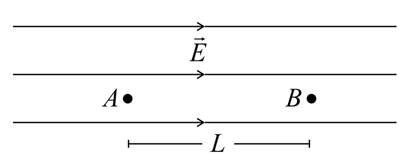
How does the quantity ΔU_{AB} defined above depend on the magnitude of the charge moved from *A* to *B*?

Use your expression above for ΔU_{AB} to define a quantity that does *not* depend on the magnitude of the charge moved from *A* to *B*. This quantity is called the *electric potential difference* between points *A* and *B*: $\Delta V_{AB} =$

What are the units of electric potential difference? This unit is called the *volt* after the Italian physicist Alessandro Volta, who is credited with the invention of the battery.

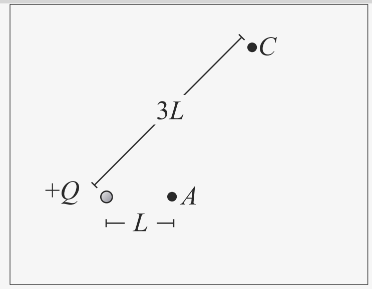
Examples

1. The diagram to the right shows two points *A* and *B* separated by a distance *L* in a region of uniform electric field $\vec{E}$. Determine the electric potential difference



ΔV_{AB}:

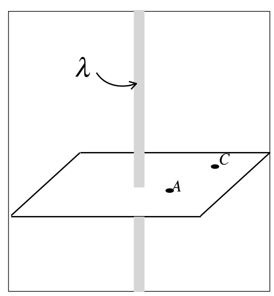
2. The diagram to the right shows two points A and C near a positive point charge $+Q$. Point A is a distance L from the charge, and point C is a distance $3L$ from the charge.
   A) On the diagram, sketch some field lines to represent the electric field.
   B) Draw and label points B and D and sketch paths such that paths AB and CD are along field lines, and paths BC and DA are perpendicular to field lines, as in the drawing on page 10.1.
   C) Use Gauss' Law to derive an expression for the magnitude of the electric field at an arbitrary distance r from the point charge. Draw your Gaussian surface on the diagram.



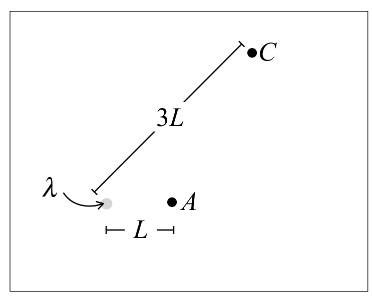
   D) Determine the electric potential difference ΔV_{AB}.

   E) Determine the electric potential difference ΔV_{AC} and explain your reasoning.

3. The diagrams to the right show two views of an infinite line of charge with linear charge density λ. Points A and C are in a plane perpendicular to the line of charge, at distances L and $3L$, respectively.
   A) On each diagram, sketch some field lines to represent the electric field in the plane containing A and C.
   B) Use Gauss' Law to derive an expression for the magnitude of the electric field at an arbitrary distance r from the line of charge. Draw your Gaussian surface on the side view.



Side view



View from above

   C) Determine the magnitude of the electric potential difference ΔV_{AC}, and state which point is at the higher potential. Clearly show the path of your integral on the view from above, and explain your reasoning.

Electric Potential

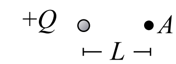
The notation for potential difference ΔV_{AB} implies a difference between two quantities V_A and V_B such that $\Delta V_{AB} = V_B - V_A$, where V_A is called the *electric potential* at *A*. We can determine the electric potential at any point once we know a point where the electric potential is zero. The electric field in a region of space is caused by some distribution of point charges, and we define our point of zero potential to be infinitely far from this distribution. In other words, we define "$V_\infty = 0$" and "$V_A = V_A - V_\infty = \Delta V_{\infty A}$." According to this definition, write the definite integral that defines V_A:

$$V_A =$$

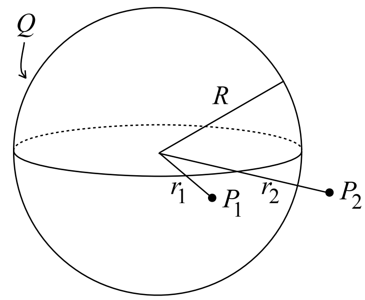
We have described the electric field $\vec{E}$ as a vector field; the magnitude of the field at a given point is the force per unit charge at that point, and the direction of the field is the direction of the force on a positive test charge at that point. Electric potential is a scalar, which has no direction,so the electric potential is a *scalar field*. Write a similar description of how to determine the electric potential at a given point in space, using the terms "work" and "infinity":

Examples

1. Consider the isolated point charge $+Q$ shown to the right. Point *A* is a distance *L* from the point charge. Use the definition above to find the electric potential at *A*.



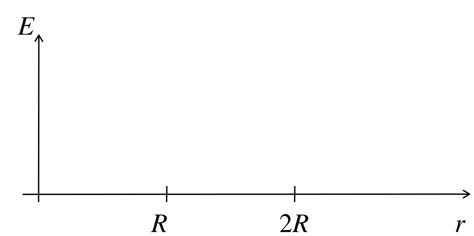
2. Consider a thin spherical shell of radius *R* with a total positive charge $+Q$ uniformly over its surface. Point P_1 is inside the sphere at a distance r_1 from the center, and P_2 is outside the sphere at a distance r_2 from the center.



   A) Use Gauss' Law to determine the magnitude of the electric field at point P_2.

   B) Use Gauss' Law to determine the magnitude of the electric field at point P_1.

   C) On the axes to the right, graph the magnitude of the electric field as a function of the distance from the center of the sphere, indicating the values of *E* at $r = R$ and $r = 2R$.

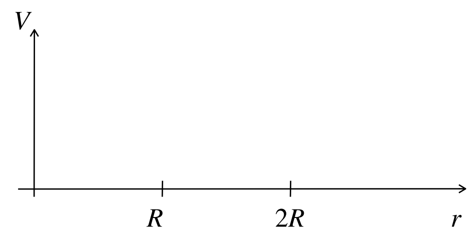


D) Determine the electric potential at point P_2.

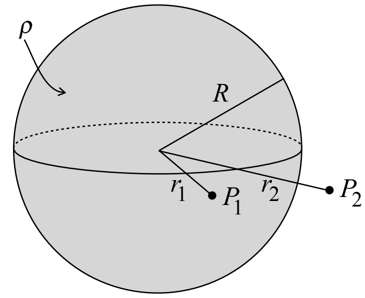
E) Determine the electric potential at point P_1.

F) What is the potential at the center of the sphere?

G) On the axes to the right, graph the electric potential as a function of the distance from the center of the sphere, indicating the values of V at $r = 0$, $r = R$ and $r = 2R$.



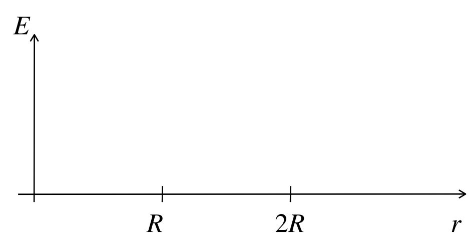
3. Consider a solid sphere of radius R with a uniform positive volume charge density ρ. Point P_1 is inside the sphere at a distance r_1 from the center, and P_2 is outside the sphere at a distance r_2 from the center.



A) Use Gauss' Law to determine the magnitude of the electric field at point P_2.

B) Use Gauss' Law to determine the magnitude of the electric field at point P_1.

C) On the axes to the right, graph the magnitude of the electric field as a function of the distance from the center of the sphere, indicating the values of E at $r = R$ and $r = 2R$.

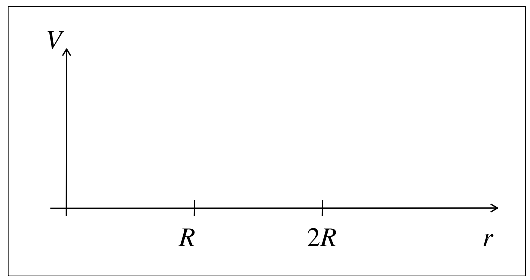


D) Determine the electric potential at point P_2.

E) Determine the electric potential at point P_1.

F) What is the potential at the center of the sphere?

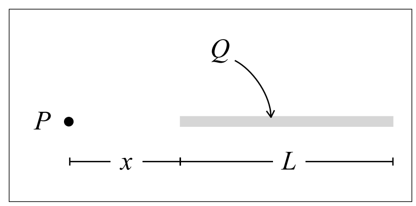
G) On the axes to the right, graph the electric potential as a function of the distance from the center of the sphere, indicating the values of V at $r = 0$, $r = R$ and $r = 2R$.



4. Consider a line of length L carrying a total positive charge Q as shown below. Point P is a distance x from the left end of the line. We can find the potential at P by adding the contributions from each part of the line, rather than integrating E.

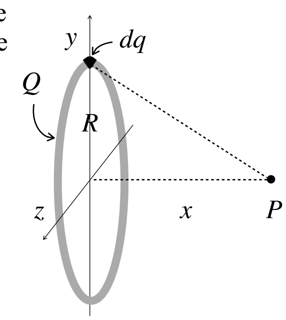
A) On the diagram, choose a small section of the line and label it dq. Choose and label parameters to determine the exact size and position of dq.

B) What is the differential contribution dV that dq makes to the potential at point P? (See example 1, page 10.4.)



C) Express dq in terms of your chosen parameters and integrate to determine the potential V_P at point P.

5. The figure to the right shows a thin ring of radius R in the y-z plane with a total positive charge Q uniformly distributed around the ring. At the top of the ring is a representative section of the ring of charge dq. Point P is on the x-axis at a distance x from the center of the ring.



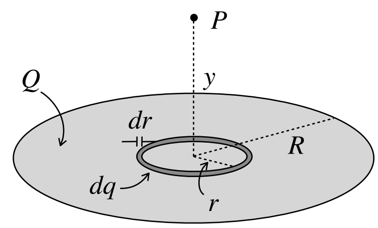
A) What is the differential contribution dV to the potential at point P?

B) Integrate to find the potential V_P at point P.

C) What is the potential at the center of the ring?

6. In reviewing examples 2, 3 and 5, a student makes the following statement: "The electric field at the center of the two spheres and at the center of the ring is zero by symmetry. Since $V = -\int \vec{E} \bullet d\vec{r}$ and $\vec{E}$ is zero, V must be zero." What's wrong with this statement?

7. The figure to the right shows a uniformly charged disk of radius R carrying a total positive charge Q. Point P is at a distance y above the center of the disk. A small ring of radius r and charge dq is shown.



A) Use the result of example 5 to express the differential contribution dV that dq makes to the potential at point P.

B) Express dq in terms of the given parameters and integrate to determine the potential V_P at point P.

C) What is the potential at the center of the disk?

Calculating $\vec{E}$ from V

Once we know the electric field $\vec{E}$, we have seen how to integrate to find the potential difference between two points in the field, or the potential at a point with respect to infinity. We can also "go backwards" and find the electric field once we know the potential. Recall the relationship between any conservative force and its potential energy function (Unit 4, page 4.10 and 4.15):

One Dimension	Three Dimensions
$F(x) = -\dfrac{dU}{dx}$	$\vec{F}(\vec{r}) = -\dfrac{\partial U}{\partial x}\hat{i} - \dfrac{\partial U}{\partial y}\hat{j} - \dfrac{\partial U}{\partial z}\hat{k}$

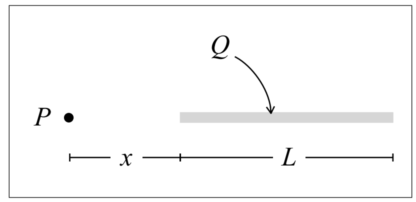
The word *gradient* is used to describe how rapidly a quantity changes with distance, just as *rate* is used to describe how a quantity changes with time. A conservative force is the *negative gradient* of its potential energy function. Since electric field is *force per unit charge* and electric potential is *electric potential energy per unit charge*, the same relationship that connects $\vec{F}$ with U will connect $\vec{E}$ with V.

Use the word "gradient" to describe the relationship between the electric field $\vec{E}$ and the electric potential V, and write the one dimensional and three dimensional equations that express this relationship:

One Dimension	Three Dimensions

Examples

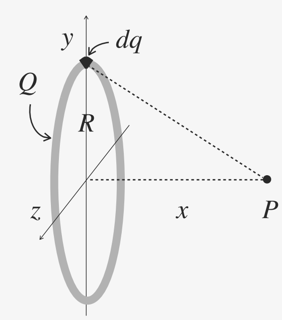
1. Consider again the line of charge from example 4, page 10.6.
   A) Use the one dimensional expression above to calculate the magnitude of the electric field at point P.



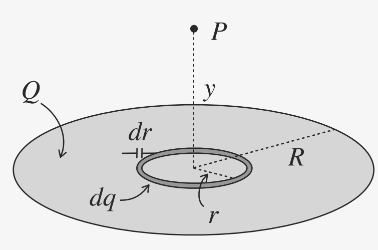
   B) Choose and label a small section dq as you did in example 4. What is the differential contribution dE at P due to this dq?

   C) Express dq in terms of your chosen parameters and integrate to determine the magnitude of $\vec{E}_P$ at point P.

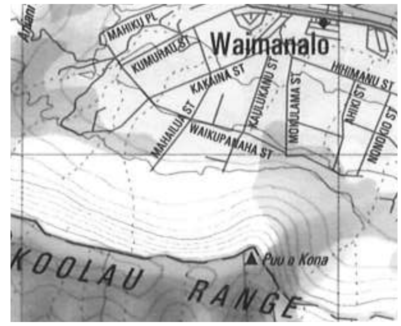
2. Consider again the ring of charge from example 5, page 10.6. Find the magnitude of the electric field at point P by calculating the potential gradient. Compare with your calculation from Unit 9, page 9.10.



3. Consider again the disk of charge from example 7, page 10.7. Find the magnitude of the electric field at point P by calculating the potential gradient. Compare with your calculation from Unit 9, page 9.12.



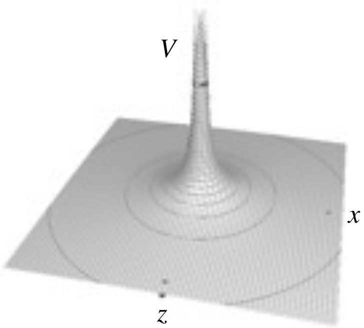
Equipotentials



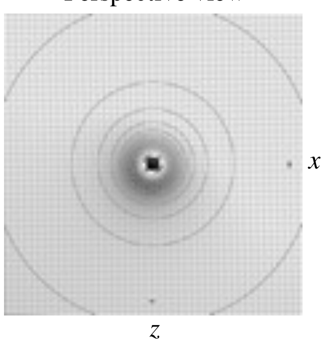
We can use a technique to visualize potential gradient that is commonly used to represent hilly terrain. The image to the right is a section of a *topographical map*, where points of equal altitude are connected by *contour lines*. What can you say about the terrain when the contour lines are close to each other?

On the analogous "map" for electric potential, the contour lines are lines of equal electric potential, and are called *equipotentials*. The 3-dimensional graphs below represent the electric potential in the *x*-*z* plane for a single positive charge, 2 positive charges, and a dipole. The *y*-axis represents the potential *V*, and equipotential contour lines are shown. You can go to the web site and download these graphs, and examine them from all angles.

Positive Charge

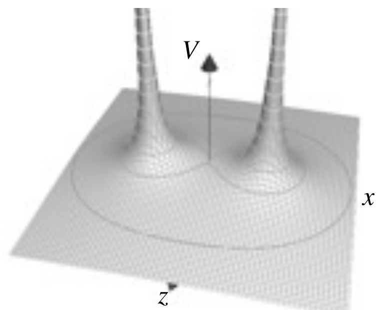


Perspective view

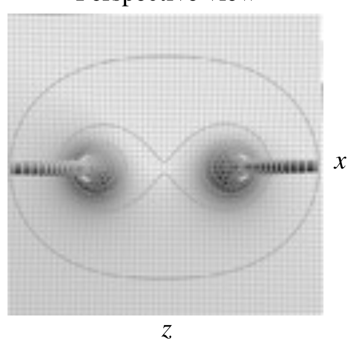


Overhead view

2 Positive Charges

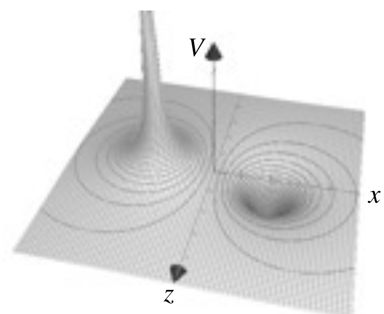


Perspective view

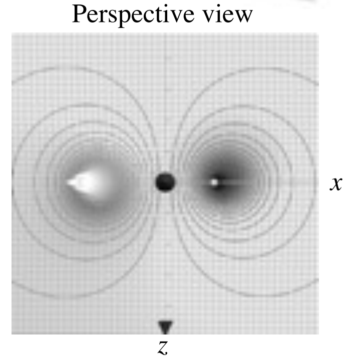


Overhead view

Dipole



Perspective view



Overhead view

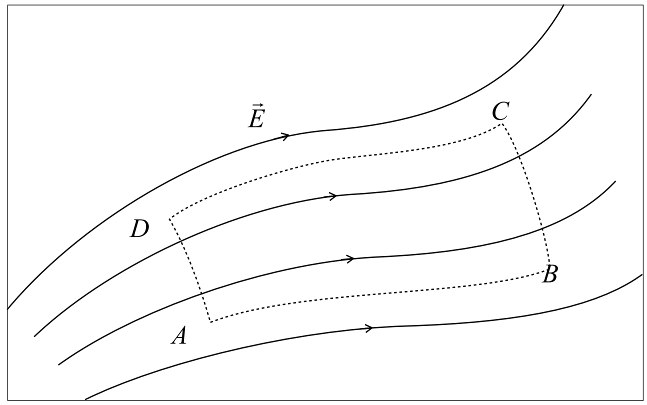
It is impossible to represent the potential in three-dimensional space, because we would need a fourth axis to represent *V*. However, we can envision the equipotentials for the above distributions by flattening the overhead views and rotating them about the *x*-axis. We see that the equipotentials are actually *surfaces* in three dimensions.

Describe the shape and relative spacing of the equipotential surfaces in the following cases:

A) A point charge

B) An infinite line of charge with constant linear charge density

C) A uniform field

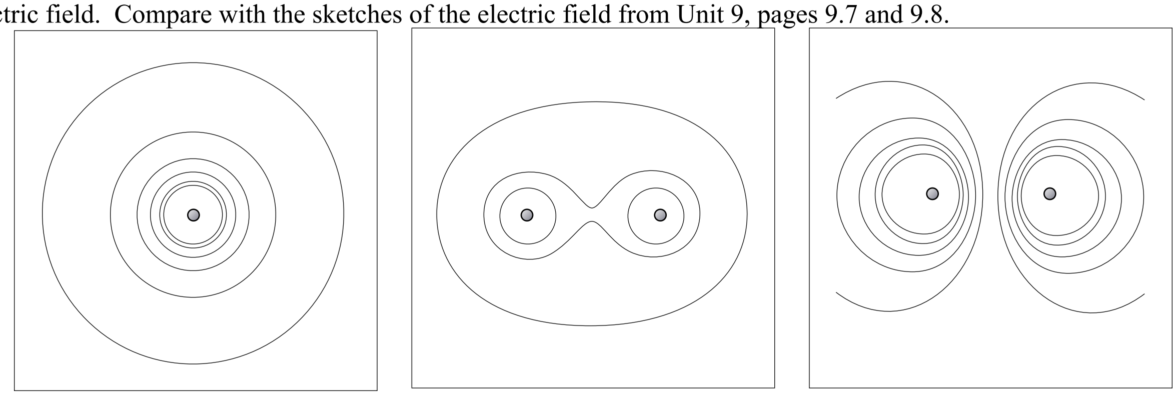


Refer again to the four paths in the drawing on page 10.1, and the work done by the electric field along each path. Which paths are equipotentials, and why?

Make a general statement relating the electric field lines and the corresponding equipotentials.

On the drawing, sketch several equipotentials in addition to the ones along the paths mentioned above. The spacing between equipotentials should be appropriate with respect to the field lines. Explain how you determine this relative spacing.

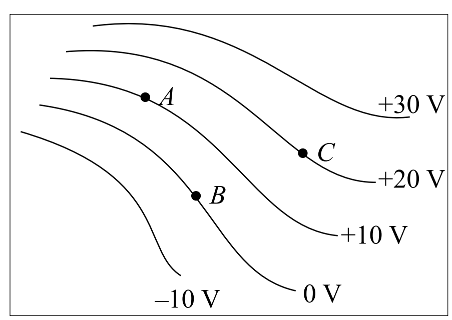
The drawings below represent the equipotentials from the previous page. The positive charge is on the left in the dipole. On the drawings, sketch several field lines in the region around the charges, and include arrowheads to indicate the direction of the electric field. Compare with the sketches of the electric field from Unit 9, pages 9.7 and 9.8.



Point Charge | 2 Positive Charges | Dipole

Consider an isolated positive point charge $q = 10^{-9}$ C. Determine the distance from this charge to the following equipotential surfaces:

The 9 V equipotential surface | The 18 V equipotential surface | The 27 V equipotential surface



The diagram to the right shows five equipotentials in a region of space.

A) On the diagram, draw a vector indicating the direction of the electric force on a proton at point *A*.

B) A proton's charge is 1.6×10^{-19} C. What is the electric potential energy of a proton at point *A*?

C) What additional information would you need in order to determine the magnitude of the force on the proton at point *A*?

D) Would there be a force on a proton if it were placed at point *B*? Explain why or why not.

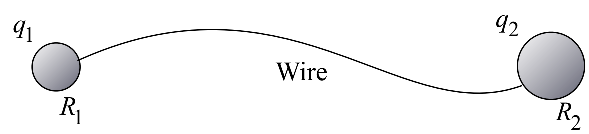
E) How much work would it take to move a proton from point *B* to point *C*? Is this work positive or negative?

Conductors

Recall that conductors have the property that charge moves very easily through them. How would you describe a piece of conducting material using the concept of equipotentials? Explain.

What can you conclude about the direction of the electric field at the surface of a charged conductor?

Example



Two isolated, spherical conductors of different radii R_1 and R_2 are held far apart and connected by a wire as shown. They have charges q_1 and q_2, respectively.

A) What is the potential at the surface of each sphere?

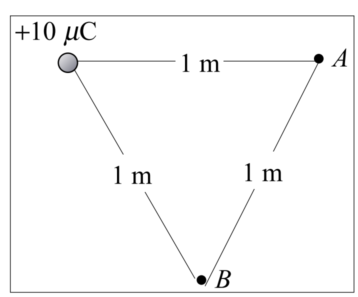
B) Determine the ratio $\frac{q_1}{q_2}$

C) Find the ratio of the surface charge densities $\frac{\sigma_1}{\sigma_2}$

Electric Potential Energy of a System of Charges

It takes work to move charges close to one another, and we have seen that the work done becomes electric potential energy. We wish to determine the electric potential energy of any system of charges.

Points *A* and *B* form an equilateral triangle with a positive 10 μC charge as shown to the right.



A) What is the potential at points *A* and *B*?

B) A positive 20 μC charge is brought from infinitely far away and placed at point *A*. How much work was needed? Was this work positive or negative?

C) What is the potential at point *B* once this charge is placed at point *A*?

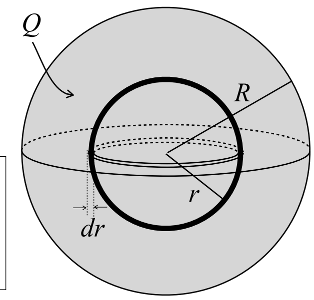
D) A negative 30 μC charge is brought from infinitely far away and placed at point *B*. How much work was needed? Was this work positive or negative?

E) What is the electric potential energy of this system of three charges? Is it positive or negative?

We can determine the electric potential energy of a distribution of charge by finding the work needed to assemble it from infinitely separated charges.

Example

A solid sphere with positive charge Q uniformly distributed throughout its volume is assembled by adding successive thin shells of radius r and thickness dr as shown to the right.



A) Determine the charge inside the shell of radius r.

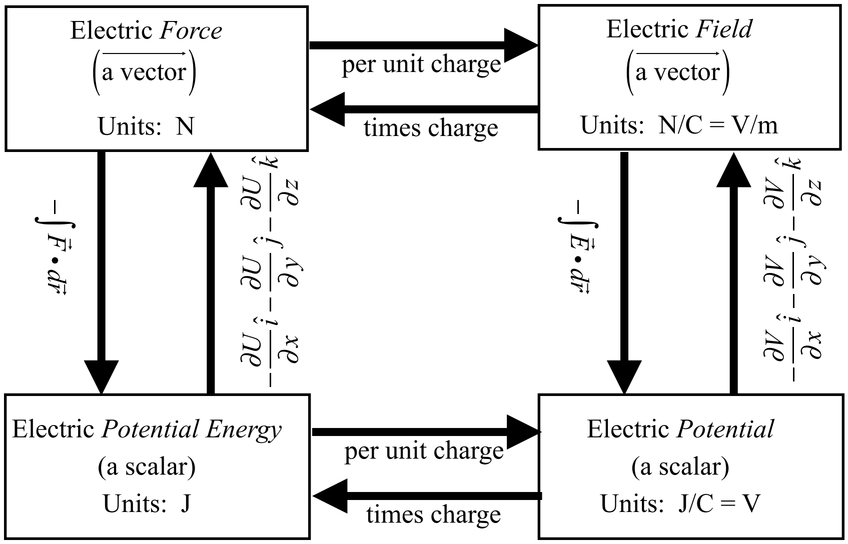
B) Determine the charge dq of the shell.

C) Determine the differential work dW needed to add the shell.

D) Integrate to find the total potential energy of the solid sphere of charge.

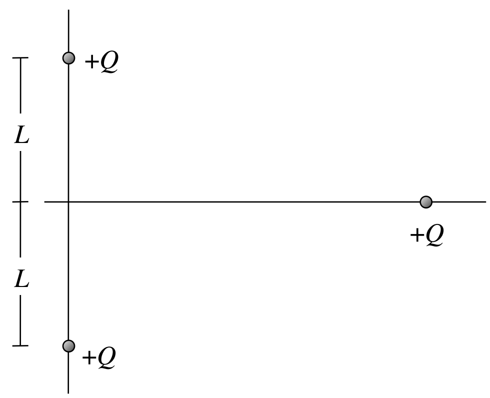
Concept Map

Below is a concept map showing the interconnections between electric *force*, *field*, *potential energy* and *potential*:



Examples

1. Two stationary point charges $+Q$ are located on the y-axis a distance $2L$ apart as shown to the right. A third charge $+Q$ is then brought in from infinity along the x-axis.

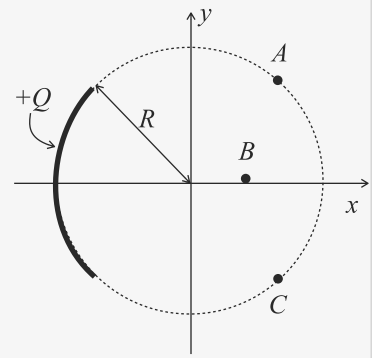


A) What is the potential energy of the system when the movable charge is at infinity?

B) Express the potential energy of the system as a function of the movable charge's position on the x-axis.

C) Assume that the movable charge is shot from infinity with some initial kinetic energy K_o. What is the minimum value of K_o such that the movable charge will pass through the center of the two fixed charges?

2. A charge +Q is uniformly distributed over a quarter circle of radius R, as shown to the right. Points A, B, and C are located as shown, with A and C located symmetrically relative to the x-axis. Express all algebraic answers in terms of the given quantities and fundamental constants.

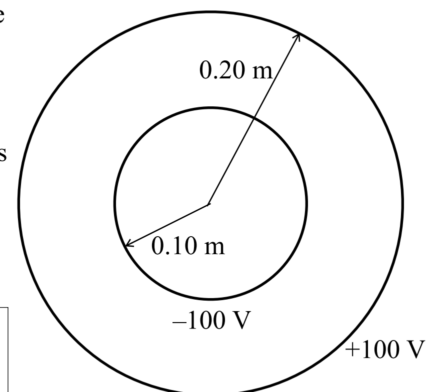


   A) Rank the magnitude of the electric potential at points A, B, and C from greatest to least. If two points have the same potential, give them the same ranking. Justify your rankings.

   B) Determine the electric potential at the origin due to the charge Q.

   C) A positive point charge q with mass m is placed at the origin and released from rest. Derive an expression for the speed of the point charge when it is very far from the origin.

3. Two thin, concentric, conducting spherical shells, insulated from each other, have radii of 0.10 m and 0.20 m, as shown to the right. The inner shell is set at an electric potential of –100 V, and the outer shell is set at an electric potential of +100 V. Let Q_i and Q_o represent the net charge on the inner and outer shells, respectively, and let r be the radial distance from the center of the shells. Express all algebraic answers in terms of Q_i, Q_o, r, and fundamental constants, as appropriate.



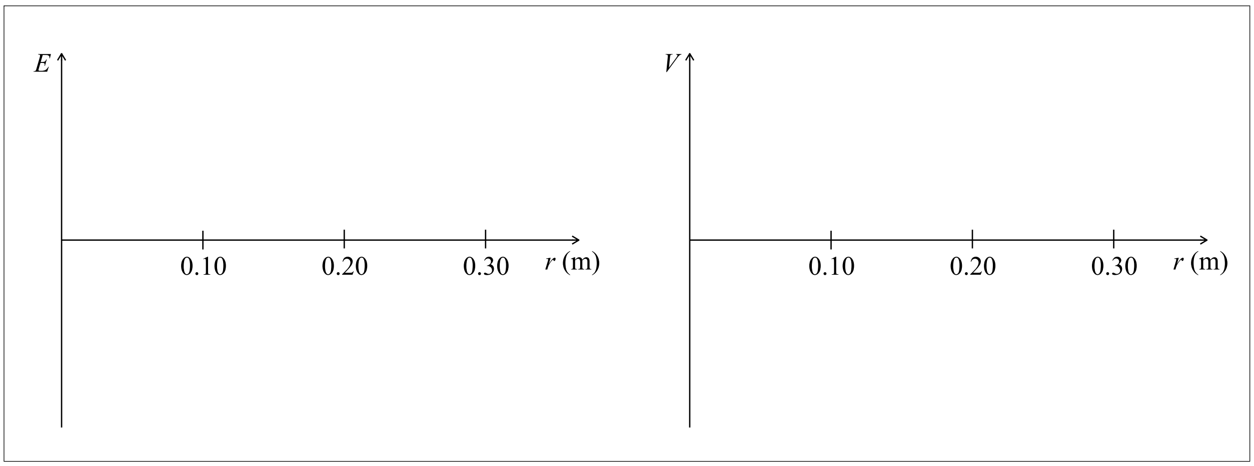
   A) Using Gauss's Law, derive an algebraic expression for the electric field $E(r)$ for 0.10 m $< r <$ 0.20 m.

   B) Determine an algebraic expression for the electric field $E(r)$ for $r >$ 0.20 m.

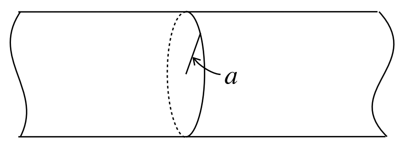
   C) Determine an algebraic expression for the electric potential $V(r)$ for $r >$ 0.20 m.

D) Using the numerical information given, calculate the value of the total charge Q_T on the two spherical shells ($Q_T = Q_i + Q_o$).

E) On the axes below, left, sketch the magnitude of the electric field E as a function of r. Let the positive direction be radially outward. On the axes below, right, sketch the electric potential V as a function of r. In both cases, indicate significant values on the vertical axis.



4. The figure at the right shows a section through a long nonconducting rod of radius a with a positive charge distributed throughout its volume. The charge distribution is cylindrically symmetric, and the total charge per unit length is λ.

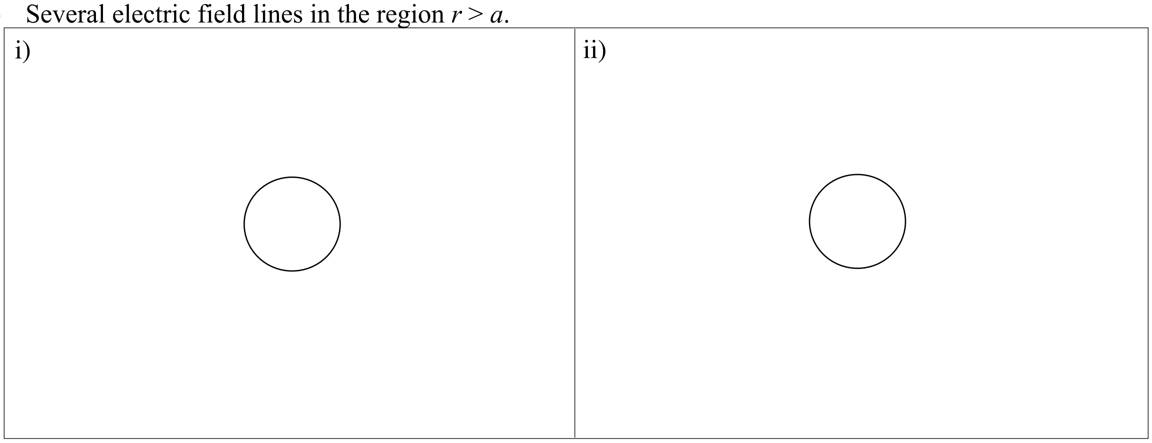


A) Use Gauss' Law to find the electric field $\vec{E}$ outside ($r > a$) the rod.

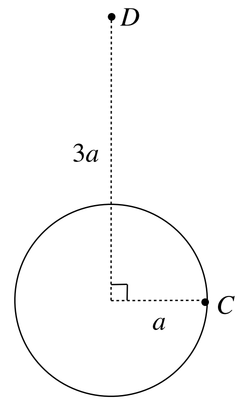
B) The diagrams below represent cross sections of the rod. On these diagrams, sketch the following:

i) Several equipotential lines in the region $r > a$.

ii) Several electric field lines in the region $r > a$.



C) In the diagram to the right, point C is a distance a from the center of the rod (ie., on the rod's surface), and point D is a distance $3a$ from the center on a radius that is 90° from point C. Determine the following:



i) The potential difference $V_C - V_D$ between points C and D

ii) The work required by an external agent to move a charge $+Q$ from rest at point D to rest at point C. State whether this work is positive or negative.

Inside the rod ($r < a$), the charge density ρ is a function of the radial distance r from the axis of the rod and is given by $\rho = \rho_o \sqrt{r/a}$, where ρ_o is a constant.

D) Determine the magnitude of the electric field E as a function of r for $r < a$. Express your answer in terms of ρ_o, a, and fundamental constants.

E) Determine the magnitude of the potential difference $V_o - V_C$ between the center of the rod and C. Express your answer in terms of ρ_o, a, and fundamental constants.

5. A spherically symmetric charge distribution has net positive charge Q_o distributed within a radius R. Its electric potential V as a function of the distance r from the center of the sphere is given by the following:

$$V(r) = \frac{Q_o}{4\pi\varepsilon_o R}\left[-2 + 3\left(\frac{r}{R}\right)^2\right] \text{ for } r < R$$

$$V(r) = \frac{Q_o}{4\pi\varepsilon_o r} \text{ for } r > R$$

Exress all algebraic answers in terms of the given quantities and fundamental constants.

A) For the following regions, derive an expression for the magnitude of the electric field $E(r)$ and state whether its direction is radially inward or radially outward:

i) $r < R$

ii) $r > R$

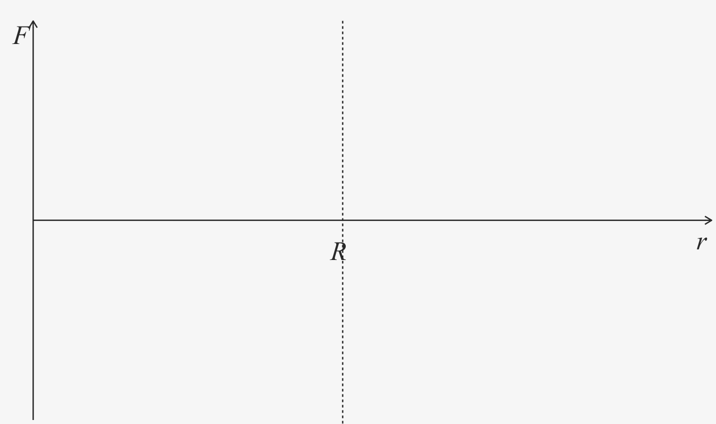
B) For the following regions, derive an expression for the enclosed charge that generates the electric field in that region, expressed as a function of r.

i) $r < R$

ii) $r > R$

C) Is there any charge on the surface of the sphere ($r = R$)? If there is, determine the charge; in either case, explain your reasoning.

D) On the axes below, sketch a graph of the force that would act on a positive test charge in the regions $r < R$ and $r > R$. Assume that a force directed radially outward is positive.



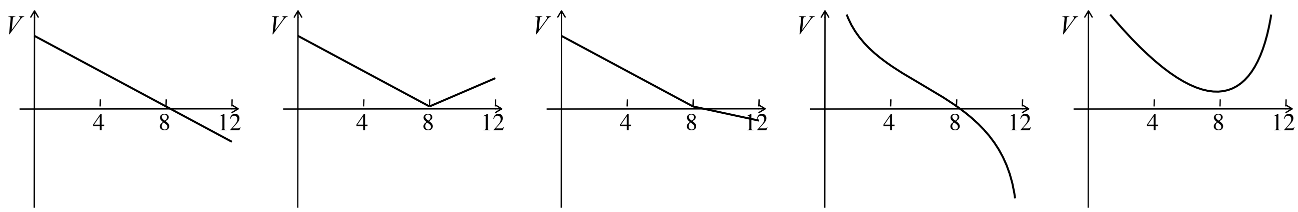
AP Physics C
Unit 10 Practice Test   Name ____________________

Part I
Multiple Choice

1. A distribution of charge is confined to a finite region of space. The difference in electric potential between any two points P_1 and P_2 due to this charge distribution depends *only* upon the

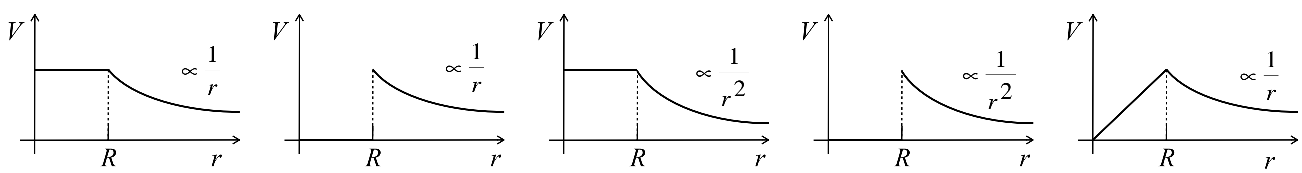
   A) charges located at the points P_1 and P_2.
   B) magnitude of a test charge moved from P_1 to P_2.
   C) value of the electric field at P_1 and P_2.
   D) path taken by a test charge moved from P_1 to P_2.
   E) none of these

2. Two small spheres having charges of +2Q and –Q are located 12 cm apart. The potential of points lying on a line joining the charges is best represented as a function of the distance x from the positive charge by which of the following?



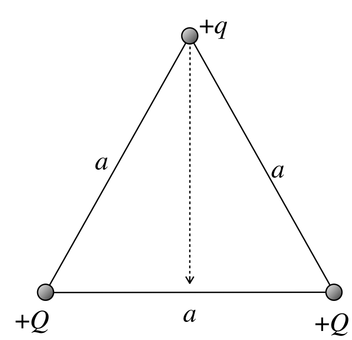
A) B) C) D) E)

3. An isolated solid spherical conductor of radius R carries a charge q. The electric potential due to this system varies as a function of the distance r from the center of the sphere in which of the following ways? (The potential is taken to be zero at infinity.)



A) B) C) D) E)

4. The two positive charges Q are fixed at the vertices of an equilateral triangle with sides of length a, as shown to the right. The work required to move $+q$ from the other vertex to the center of the line joining the fixed charges is

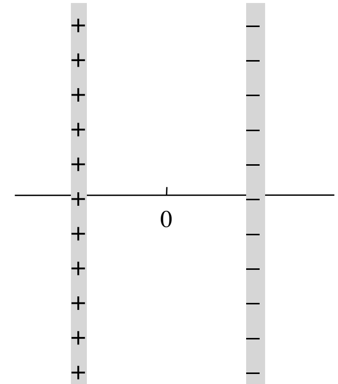


   A) zero
   B) $\frac{kQq}{a}$
   C) $\frac{kQq}{a^2}$
   D) $\frac{2kQq}{a}$
   E) $\frac{\sqrt{2}kQq}{a}$

5. Positive charge is distributed uniformly throughout a non-conducting sphere. The highest electrical potential occurs

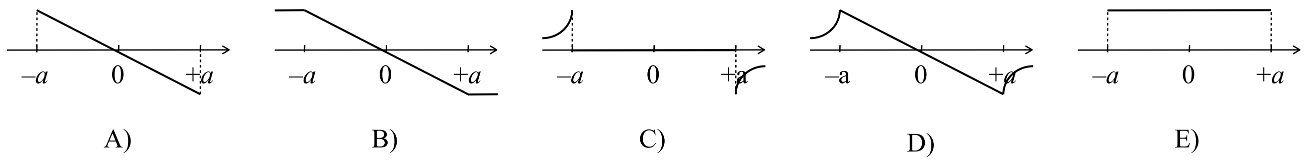
   A) at the center
   B) at the surface
   C) halfway between the center and the surface
   D) just outside the surface
   E) far from the sphere

For questions 6 and 7: Two infinite parallel sheets of charge perpendicular to the x-axis have equal and opposite charge densities as shown to the right. The sheet that intersects at $x = -a$ has uniform positive surface charge density; the sheet that intersects at $x = +a$ has uniform negative surface charge density. Consider the graphs drawn below in answering questions 6 and 7.



6. Which graph best represents the plot of electric field as a function of x?

7. Which graph best represents the plot of electric potential as a function of x?



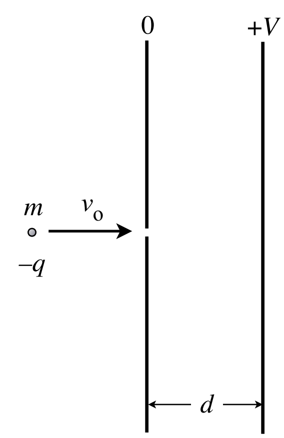
8. A 2 meter long stick is parallel to a uniform 200 N/C electric field. The potential difference between its ends is

   A) zero
   B) 1.6×10^{-17} V
   C) 3.2×10^{-17} V
   D) 100 V
   E) 400 V

9. A conducting sphere with a radius of 0.10 m has 1.0×10^{-9} C of charge on it. The magnitude of the electric field just outside the surface of the sphere is

   A) zero
   B) 450 V/m
   C) 900 V/m
   D) 4,500 V/m
   E) 90,000 V/m

10. As shown in the diagram to the right, a charged particle having a mass m and charge $-q$ is projected into the region between two large parallel plates with a speed v_o to the right. The potential difference between the plates is V and they are separated by a distance d. What is the net change in kinetic energy of the particle during the time it takes the particle to traverse the distance d?

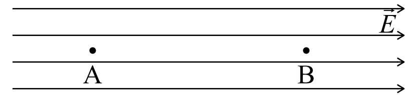


   A) $+\frac{1}{2}mv_o^2$
   B) $-\frac{qV}{d}$
   C) $+\frac{2qV}{mv_o^2}$
   D) $+qV$
   E) None of the above

11. Which of the following statements is correct?

   A) A proton tends to go from a region of low electrical potential to a region of high electrical potential.
   B) The potential of a negatively charged conductor must be negative.
   C) If $\vec{E} = 0$ at point P, then V must be zero at P.
   D) If $V = 0$ at point P, then $\vec{E}$ must be zero at P.
   E) None of the above statements are correct.

12. An electron moves from point A to point B in the direction of a uniform electric field. During this displacement:



A) The work done by the field is positive and the potential energy of the electron-field system increases.
B) The work done by the field is negative and the potential energy of the electron-field system increases.
C) The work done by the field is positive and the potential energy of the electron-field system decreases.
D) The work done by the field is negative and the potential energy of the electron-field system decreases.
E) The work done by the field is positive and the potential energy of the electron-field system does not change.

13. An electron is accelerated from rest through a potential difference V. Its final speed is proportional to

A) V

B) V^2

C) $\sqrt{V}$

D) $\dfrac{1}{V}$

E) $\dfrac{1}{\sqrt{V}}$

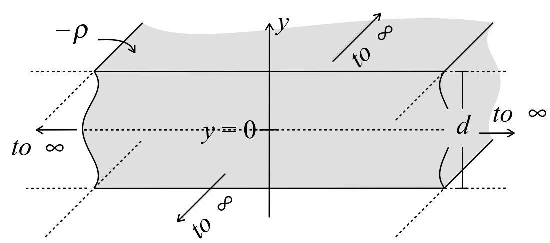
14. In separate experiments, four different particles each start from far away with the same speed directly at a gold nucleus. The masses and charges of the particles are:

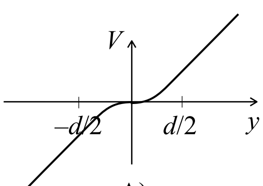
Particle 1: mass m, charge q
Particle 2: mass $2m$, charge $2q$
Particle 3: mass $2m$, charge $q/2$
Particle 4: mass $m/2$, charge $2q$

Rank the particles according to the distance of closest approach to the gold nucleus, from smallest to largest.

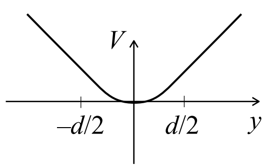
A) $1 < 2 < 3 < 4$
B) $4 < 3 < 2 < 1$
C) $3 < 1 = 2 < 4$
D) $4 < 1 = 2 < 3$
E) $1 = 2 < 3 < 4$

15. An insulating slab of infinite length and width has thickness d. The slab has a uniform negative volume charge density $-\rho$. The positive y direction is up, and the origin of the y axis is at the center of the slab. If the potential at the center of the slab is taken to be zero, then the potential as a function of y is best represented by

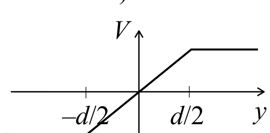




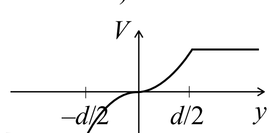
A)



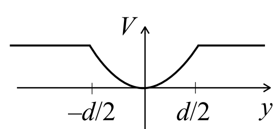
B)



C)



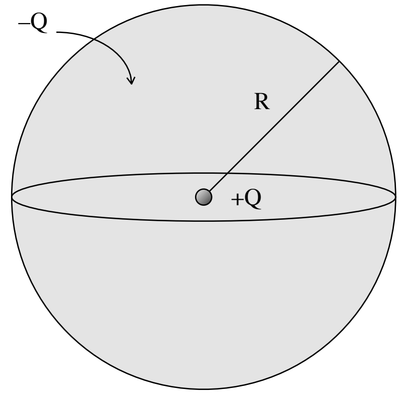
D)



E)

1.E 2.D 3.A 4.D 5.A 6.E 7.B 8.E 9.C 10.D 11.E 12.B 13.C 14.C 15.B

Part II
Show your work
Credit depends on the quality and clarity of your explanations



1. A negative charge $-Q$ is uniformly distributed throughout the spherical volume of radius R shown to the right. A positive point charge $+Q$ is at the center of the sphere. Determine each of the following in terms of the quantities given and fundamental constants.

   A) The electric field E outside the sphere at a distance $r > R$ from the center
   B) The electric potential V outside the sphere at a distance $r > R$ from the center
   C) The electric field inside the sphere at a distance $r < R$ from the center
   D) The electric potential inside the sphere at a distance $r < R$ from the center

2. Three particles A, B, and C have equal positive charges Q and are held in place at the vertices of an equilateral triangle with sides of length L, as shown in the figures to the right. The dotted lines represent the bisectors for each side. The base of the triangle lies on the x-axis, and the altitude of the triangle lies on the y-axis.

   A) i) Point P_1, the intersection of the three bisectors, locates the geometric center of the triangle and is one point where the electric field is zero. On Figure 1 to the right, draw the electric field vectors $\vec{E}_A$, $\vec{E}_B$, and $\vec{E}_C$ at P_1 due to each of the three charges. Be sure your arrows are drawn to reflect the relative magnitude of the fields.

   ii) Another point where the electric field is zero is point P_2 at $(0, y_2)$. On Figure 2 to the right, draw electric field vectors $\vec{E}_A$, $\vec{E}_B$, and $\vec{E}_C$ at P_2 due to each of the three point charges. Indicate in the table below whether the magnitude of each of these vectors is greater than, less than, or the same as for point P_1.

	Greater	Less	The same
$\vec{E}_A$			
$\vec{E}_B$			
$\vec{E}_C$			

   B) Explain why the x-component of the total electric field is zero at any point on the y-axis.

   C) Write a general expression for the electric potential V at any point on the y-axis inside the triangle in terms of Q, L, and y.

   D) Describe how the answer to part C) could be used to determine the y-coordinates of points P_1 and P_2 at which the electric field is zero. (You do not need to actually determine these coordinates.)

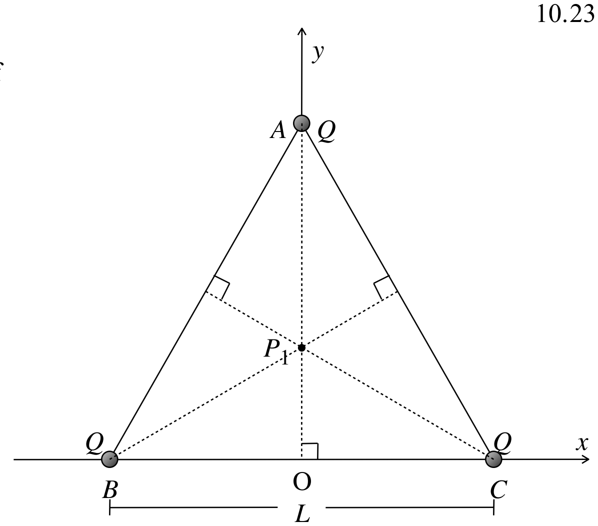


Figure 1

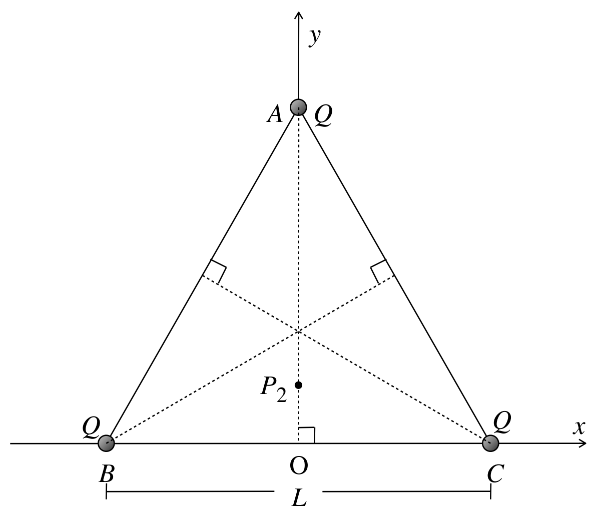
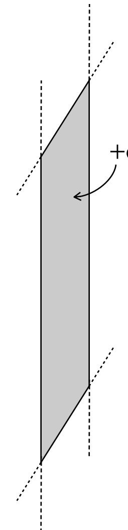


Figure 2

AP Physics C
Unit 11 Name____

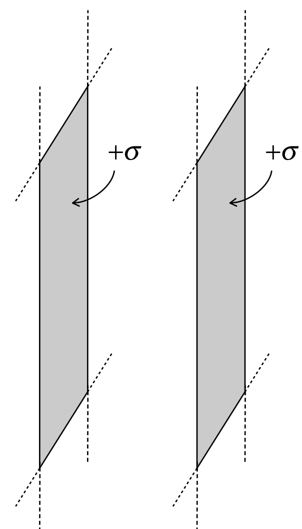
Capacitance

The diagram to the right represents an infinite sheet with uniform positive charge density $+\sigma$. Use Gauss' Law to find the magnitude and direction of the electric field $\vec{E}$ on both sides of the sheet. Be specific about your choice of Gaussian surface, and how you use symmetry.



The next diagram represents two identical sheets parallel to each other.

A) At what point(s) can you use symmetry to show that the electric field is zero? Explain.



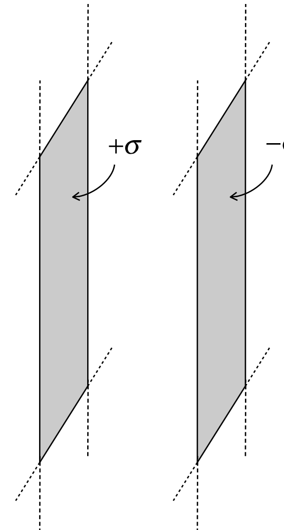
Use Gauss' Law to find the magnitude and direction of the electric field $\vec{E}$ in the following regions. Be specific about your choice of Gaussian surface, and how you use symmetry.

B) anywhere on either side of the two sheets

C) anywhere between the two sheets

D) Show that the results from parts B) and C) are consistent with the fact that the net electric field at any point is the vector sum of the electric fields due to each part of the charge distribution. This principle is called *superposition*.

Consider two infinite sheets with equal but opposite charge densities as shown to the right. Use the principle of superposition to determine the magnitude and direction of the electric field $\vec{E}$ in the following regions:

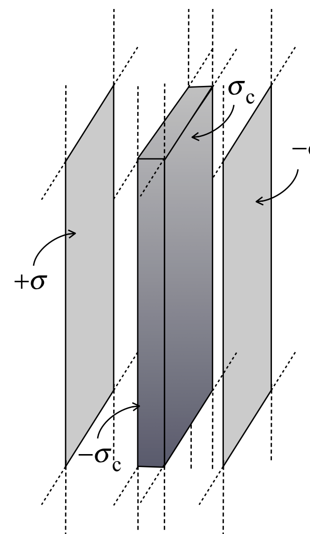


A) anywhere to the left of the left sheet

B) anywhere to the right of the right sheet

C) anywhere between the sheets

An uncharged conducting plate is inserted between the two sheets. Electrical forces separate the charges in the conducting plate toward the oppositely charged sheets, so that at equilibrium, the left side of the plate has negative charge density $-\sigma_c$ and the right side has positive charge density $+\sigma_c$. Effectively, there are now four sheets.



A) Use the principle of superposition to determine the following:
   i) the magnitude of the electric field between the left sheet and the plate

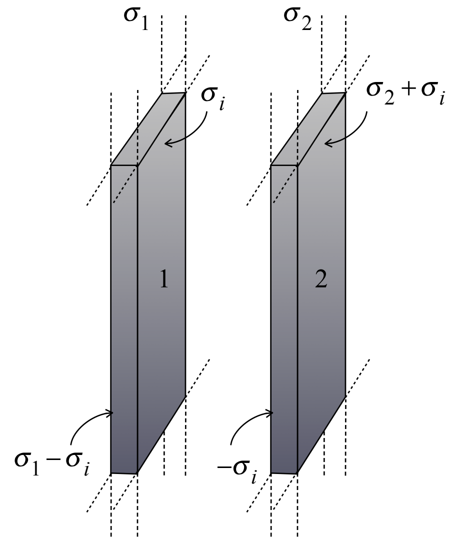
   ii) the magnitude of the electric field between the plate and the right sheet

B) Use Gauss' Law to determine the following. Be specific about your choice of Gaussian surface:
   i) the magnitude of the electric field to the left of the left sheet and to the right of the right sheet

   ii) the charge density σ_c on the surfaces of the conductor in terms of σ.

11.3

Two infinite conducting plates 1 and 2 are parallel to each other, and carry different total charge densities σ_1 and σ_2, as shown. For simplicity, assume that both charge densities are positive and that $\sigma_1 > \sigma_2$. Electric forces separate the charges so that they settle on the surfaces of the plates, effectively making four infinite sheets of charge.



A) The charge densities on the inner surfaces are shown as σ_i and $-\sigma_i$.
Use Gauss' Law to justify the fact that these charge densities are equal and opposite. Be specific about your choice of Gaussian surface.

B) The electric field inside each conducting plate is zero. Use the principle of superposition to find the charge density σ_i in terms of σ_1 and σ_2.

C) Determine the charge densities in the following regions in terms of σ_1 and σ_2:

Left side of 1	Right side of 1	Left side of 2	Right side of 2

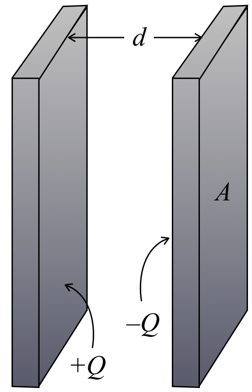
D) In the case that the charge densities are equal and of the same sign ($\sigma_1 = \sigma_2$), determine the charge densities in the following regions in terms of σ_1:

Left side of 1	Right side of 1	Left side of 2	Right side of 2

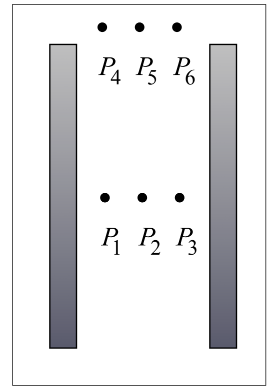
E) In the case that the charge densities are equal and of opposite sign ($\sigma_1 = -\sigma_2$), determine the charge densities in the following regions in terms of σ_1:

Left side of 1	Right side of 1	Left side of 2	Right side of 2

Infinite sheets and plates are hard to come by; in reality we find objects of finite size. For example, consider two conducting plates of finite area A separated by a distance d. Our previous discussion is valid provided the area is large compared to d; the diagram to the right is exaggerated for clarity.



Perspective view



Edge view

Imagine that each plate is initially uncharged, and we remove a charge $+Q$ from the right plate and place it on the left plate, leaving the right plate with a net charge of $-Q$. We have seen that the charges will settle on the inside surfaces of the plates. In the edge view to the right, points P_1 through P_3 are along the center line of the plates, and points P_4 through P_6 are just above the top of the plates.

A) On the diagram, draw small vectors at each of these points to indicate the direction of the electric field due to the charge on the plates.
B) On the diagram, sketch several electric field lines between the plates to show how the field varies from the top to the bottom of the charged plates.

When the separation distance is small, these "edge effects" are negligible, and we can assume that the electric field is uniform between the plates, and stops abruptly at the edges. We will neglect edge effects in what follows. In terms of the quantities in the diagram:

C) What is the charge density on the surface of the left plate? $\sigma =$

D) What is the magnitude of the electric field between the plates? $E =$

E) What is the potential difference between the plates? $V =$

F) What is the ratio $\frac{Q}{V}$? $\frac{Q}{V} =$

Notice that the ratio $\frac{Q}{V}$ depends only on the dimensions of the pair of plates, and nothing else. This ratio is called the *capacitance*, and this pair of plates is called a *parallel-plate capacitor*. We can think of capacitance as the *charge-carrying capacity per volt*. What are the SI units of capacitance? This unit is called the *Farad* (F) after Michael Faraday. (Note: You are expected to know or be able to quickly derive the formula for the capacitance of a parallel-plate capacitor.)

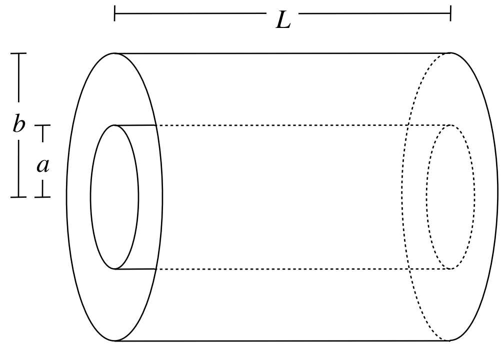
Any two conductors separated by an insulator can behave as a capacitor. We can remove charge from one of them and place it on the other, and there will be a potential difference between them. The potential difference will be proportional to the amount of charge we have moved. Whether or not the conductors are flat, we call them "plates."

Given information about the size, shape and separation distance of the capacitor plates, we calculate the capacitance by taking the following steps:

1. Assume each plate has a charge Q, and use Gauss' Law to find the field $\vec{E}$.
2. Integrate to find the potential difference between the plates ($V = -\int \vec{E} \cdot d\vec{r}$).
3. Use the definition of capacitance to find C.

Examples

1. A capacitor is made from two concentric cylindrical conductors of radii a and b and length L as shown to the right. Let the plates be charged with a charge Q (ie., one has a charge $+Q$ and the other has a charge $-Q$).

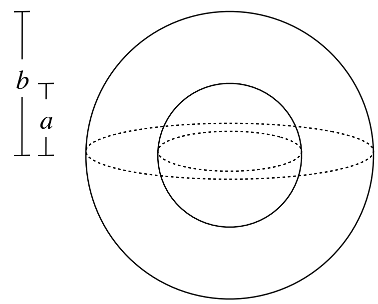


   A) Use Gauss' Law to find $\vec{E}$ between the plates (ie., at a point a distance r from the central axis, where $a < r < b$).

   B) Determine the potential difference between the plates.

   C) Determine the capacitance of the cylindrical capacitor.

2. A capacitor is made from two concentric spherical conductors of radii a and b as shown to the right. Let the plates be charged with a charge Q.

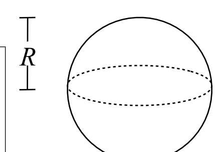


   A) Use Gauss' Law to find $\vec{E}$ between the plates ($a < r < b$).

   B) Determine the potential difference between the plates.

   C) Determine the capacitance of the spherical capacitor.

   D) An isolated spherical conductor of radius R can be regarded as a capacitor, with the other "plate" at infinity. Use the result above to find the capacitance of such a capacitor.



Energy Stored in a Capacitor

Charging a capacitor can be thought of as a continuous process of removing a small charge dq from one plate and placing it on the other plate. This becomes harder and harder as the charge builds up on the plates. The work it takes to charge the capacitor with a total charge Q is stored as potential energy.

Let the capacitance of the capacitor be C. At some time t during this process, the charge has built up to $q(t)$.

A) What is the potential difference between the plates at time t? $V(t) =$

B) What is the differential work dW required to move the next dq? $dW =$

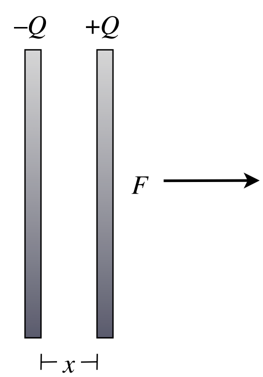
C) Integrate with appropriate limits to determine the energy U_C stored in the fully charged capacitor.

D) If the potential difference between the plates of the fully-charged capacitor is V, express U_C in terms of

i) C and V: $U_C =$ ii) Q and V: $U_C =$

Example

The plates of an isolated parallel-plate capacitor are pulled apart slowly by a force F. Each plate has charge Q and area A. Neglect edge effects and express the following in terms of x, A, Q and constants:



A) the capacitance C

B) the energy U_C stored in the capacitor

C) the change in capacitance dC as x is increased by dx.

D) the change in stored energy dU_C in the capacitor as x is increased by dx.

E) the force F required to separate the plates (Hint: How are F and dU_C related?)

F) the magnitude of the electric field at the right plate due to the charge on the left plate

G) the force on the right plate due to this field

Energy Density

The energy stored in a capacitor is due to the electric field created by its charge. We can think of the energy as actually being stored in the electric field, rather than the capacitor. We define the *electric energy density* u_E in a region of space as the *energy per unit volume* in that region.

A) A parallel-plate capacitor has plates of area A and separation distance d. It is charged to a potential difference V. Determine the energy density in the region between the capacitor plates.

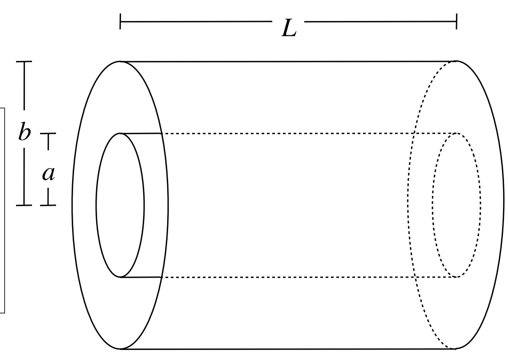
B) Express the electric energy density u_E in terms of the magnitude of the electric field E between the plates.

The electric energy density u_E depends only on the strength of the electric field, and on nothing else.

Examples

1. The cylindrical capacitor to the right (see page 11.5) has a charge Q.

A) What is the electric energy density u_E between the plates at a distance r from the axis?

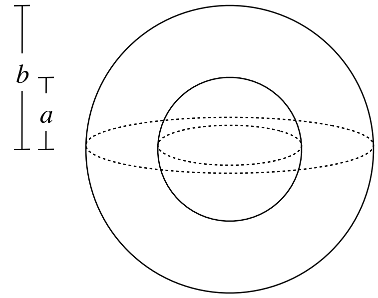


B) Consider a thin cylindrical shell between the plates, of radius r, thickness dr, and length L. What is the differential energy dU stored in the electric field of this shell?

C) Integrate with appropriate limits to find the energy contained in the electric field of the space between the plates.

D) Use your expression for the capacitance of this capacitor (page 11.5) and the energy stored in a capacitor (part C), page 11.6) to determine the energy stored in the cylindrical capacitor.

2. The spherical capacitor to the right (see page 11.5) has a charge Q.



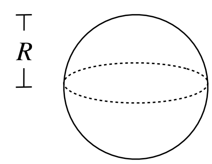
A) What is the electric energy density u_E between the plates at a distance r from the center?

B) Consider a thin spherical shell of radius r and thickness dr. What is the differential energy dU stored in the electric field of this shell?

C) Integrate with appropriate limits to find the energy contained in the electric field of the space between the plates.

D) Use your expression for the capacitance of this capacitor (page 11.5) and the energy stored in a capacitor (part C), page 11.6) to determine the energy stored in the spherical capacitor.

E) Adjust your limits from the integration in part C) to determine the energy contained in the electric field in all of the space surrounding an isolated spherical conductor of radius R carrying a charge Q.



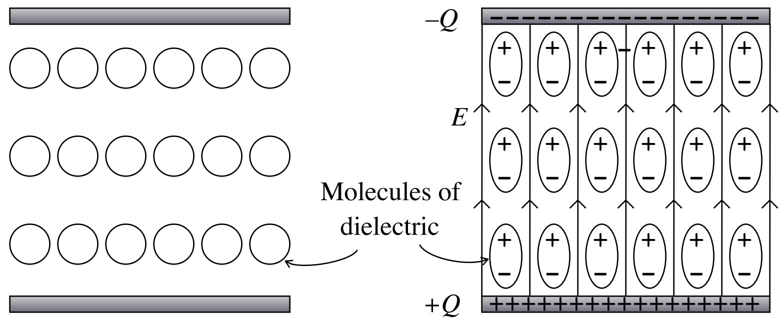
F) At some point in charging this isolated spherical conductor, it has a total charge $q(t)$. What is the potential of the sphere at this time in terms of q and R? $V(t) =$ What is the differential work needed to add another dq? $dW =$

G) Integrate with appropriate limits to determine the total work required to charge the sphere with a charge Q. Compare with your answer to part E).

Dielectrics

The insulator that separates the plates of a capacitor may be made of a material that can be polarized by an electric field. Such materials are called *dielectrics*. The molecules of the dielectric remain in place when the electric field is applied, but the centers of positive and negative charge are pulled in opposite directions by the field.

Near the surface of the negatively charged plate, there is a layer of positive charge, and the reverse happens near the positively charged plate, with the result that the net electric field between the plates is reduced.



uncharged   charged

A) If you integrate to find the potential difference V between the plates, will the result be *greater*, *less* or *the same* as it would be without the dielectric? Explain.

B) Is the capacitance of the capacitor *greater*, *less* or *the same* as it would be without the dielectric? Explain.

Regardless of the shape of the capacitor, the introduction of a dielectric between the plates has an effect that depends only on the properties of the dielectric. The *dielectric constant* κ (kappa) indicates this effect. It is defined as:

$$\kappa = \frac{C_{\text{with dielectric}}}{C_{\text{with vacuum}}}$$

There is a table of dielectric constants at http://hyperphysics.phy-astr.gsu.edu/hbase/tables/diel.html.

C) By definition, then, what is the dielectric constant of a vacuum?

D) Could a dielectric be *less than* the dielectric of a vacuum? Explain.

Example

A parallel plate capacitor with capacitance C_o is charged to a potential difference V_o. A dielectric slab with constant κ is then slipped in between the plates, completely filling the gap.

A) What is the new capacitance C?

B) What is the new potential difference V?

C) What was the initial energy U_o?

D) What is the energy U with the dielectric?

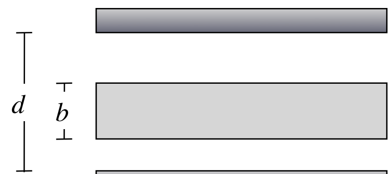
E) What fraction of the initial energy has been lost?

Gauss' Law with Dielectric

The presence of dielectric material "artificially" affects the electric field, which would affect the flux calculated in Gauss' Law. Therefore, if a portion of our Gaussian surface passes through dielectric material, the flux has to be *multiplied* by κ, because the electric field is artificially *reduced* by κ. So for that portion the contribution to the total flux is $\int \kappa \vec{E} \bullet d\vec{A}$.

Example

The figure to the right shows a parallel-plate capacitor of plate area A and plate separation d. The plates are charged to a potential difference V_o. Later, a dielectric slab of thickness b and dielectric constant κ is placed somewhere between the plates as shown. Express your answers to the following in terms of A, d, V_o, b and κ.



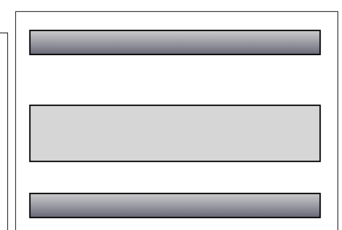
A) What is the capacitance C_o before the slab is inserted?

B) How much charge is on the plates?

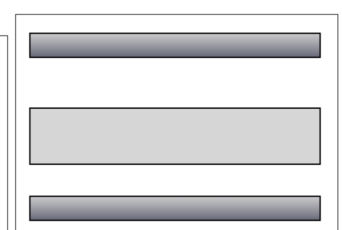
C) What is the magnitude of the electric field E_o between the plates before the slab is introduced?

The dielectric slab is placed between the plates. Express the following in terms of E_o and κ.

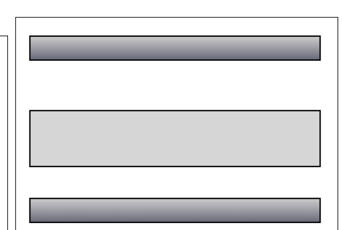
D) Use Gauss' Law to find the magnitude of the electric field E_1 in the gap between the upper plate and the dielectric slab. Draw your Gaussian surface on the figure to the right.



E) Use Gauss' Law to find the magnitude of the electric field E_2 in the gap between the lower plate and the dielectric slab. Draw your Gaussian surface on the figure to the right.



F) Use Gauss' Law to find the magnitude of the electric field E_3 inside the dielectric slab. Draw your Gaussian surface on the figure to the right.



G) Calculate the potential difference between the plates after the slab is introduced. Express in terms of V_o, b, d and κ.

H) Calculate the capacitance with the slab in place.

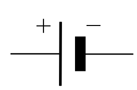
I) Does your expression make sense when $b = 0$ and when $b = d$? Explain.

J) If the slab were a *conductor* rather than a dielectric (insulator), what would be different in the steps above?

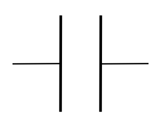
K) Find the capacitance with the *conducting* slab in place.

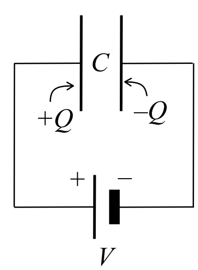
Capacitors in a Circuit

In practice, the process of charging a capacitor takes place in a circuit, with a battery supplying the energy needed to separate the charges onto the plates of the capacitor. In this context, we care only about the capacitance C of the capacitor, and not its shape or size, and we care only about the potential difference V supplied by the battery, not its chemical mechanism.



Battery

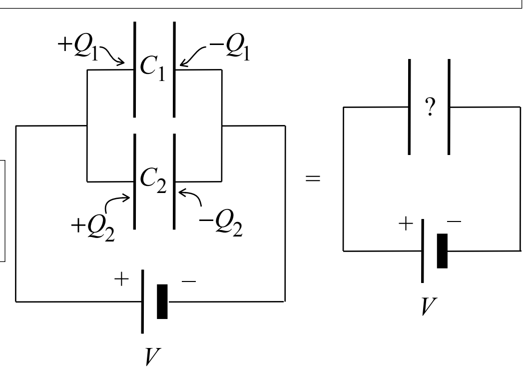

Capacitor



Circuit

A circuit with a single capacitor connected to a battery is shown to the right. When the capacitor is fully charged, the potential difference between its plates is the same as that of the battery. Use the fact that a wire is a conductor to explain why this is so.

Multiple capacitors can be connected in a circuit in a variety of ways. In the diagram to the right, the left plates of the two capacitors are at the same potential and the right plates are at the same potential. Explain why this is so.



Capacitors connected such that their plates are at the same potential in this way are connected *in parallel*. We wish to find the equivalent capacitance C of the pair.

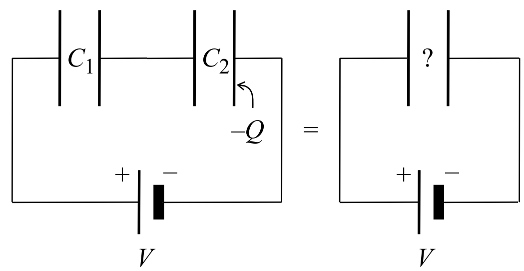
A) What is the total charge Q that has been moved from the right "plate" of the equivalent capacitor to the left "plate"?

$Q =$

B) Use the definition of capacitance to express this equation in terms of the appropriate C's and V's:

C) *Parallel Rule*: Express the equivalent capacitance C of the pair in terms of C_1 and C_2:

Capacitors connected such that the potential differences across each pair of plates is not necessarily the same, as shown to the right, are connected *in series*. We wish to find the equivalent capacitance C of the pair.



A) A charge $+Q$ is removed from the right plate of C_2 and moved through the battery, leaving this plate with a net charge $-Q$. Onto which of the other three plates does this $+Q$ charge go?

B) Explain why this charge doesn't go to the other two plates.

C) Use Gauss' Law for each capacitor to determine the charge that *does* go onto the other two plates.

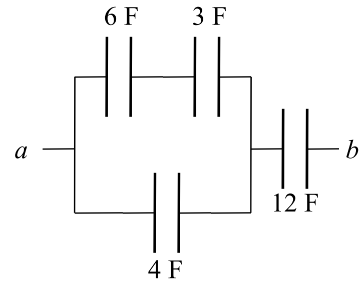
D) Let the potential difference between the plates of the capacitors be V_1 and V_2, respectively. What is the potential difference across the combined capacitor, from the left plate of C_1 to the right plate of C_2? How is this related to the potential difference V of the battery?

E) Use the definition of capacitance to express this equation in terms of the appropriate C's and Q's:

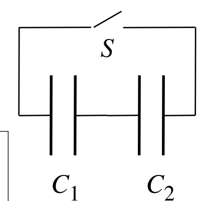
F) *Series Rule*: Express the equivalent capacitance C of the pair in terms of C_1 and C_2:

Examples

1. Use the series and parallel rules to determine the total capacitance between points a and b in the circuit segment to the right. Show how you are using each rule in turn.



2. A capacitor with capacitance C_1 is charged to an initial potential difference V_o using a battery. The capacitor is then connected as shown to the right to another, initially uncharged, capacitor C_2.



   A) When the switch S is closed, are the capacitors connected in series or parallel? Explain your choice.

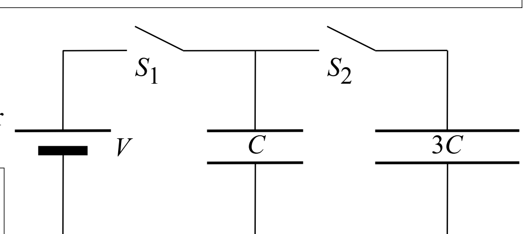
   B) Find the potential difference across each capacitor after the switch is closed, in terms of V_o, C_1 and C_2.

   C) How much energy did the battery initially supply to charge C_1?

   D) How much energy is stored in the capacitors after the switch is closed?

   E) What fraction of the initial energy was lost?

3. In the circuit shown to the right, both switches are initially open. The capacitor on the left has a capacitance C, and the capacitor on the right has a capacitance $3C$. The battery has a voltage V.

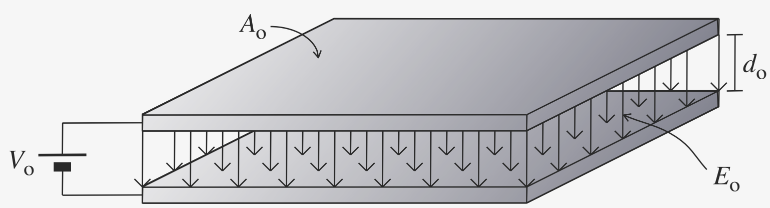


   A) First, switch S_1 is closed. Determine the charge on the left capacitor when equilibrium is reached.

B) Next, switch S_1 is opened, then S_2 is closed. Determine the charge on the left capacitor when equilibrium is again reached.

C) S_2 remains closed, and now S_1 is also closed. How much *additional* charge flows through the battery?

4. A parallel-plate capacitor is formed from plates of area A_o, separated by a distance d_o, giving it a capacitance C_o. The capacitor is connected to a battery of voltage V_o, creating an electric field of magnitude E_o, and as a result acquires a charge Q_o, surface charge density σ_o, and energy U_o. Answer the following in terms of these quantities.



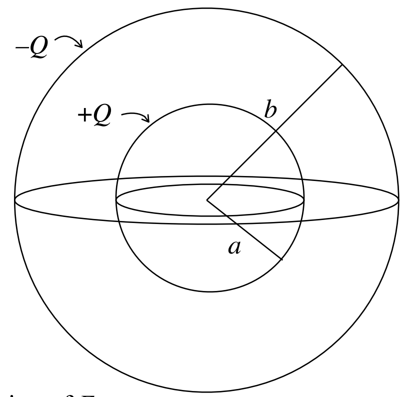
A) The plate area is doubled to $2A_o$ while the plates remain connected to the battery. What are the new values of V, C, q, σ, E, and U? Explain your reasoning.

B) The separation distance of the original capacitor is doubled to $2d_o$, while the plates remain connected to the battery. What are the new values of V, C, q, σ, E, and U? Explain your reasoning.

C) The battery is disconnected from the original capacitor, and its plate area is doubled to $2A_o$. What are the new values of V, C, q, σ, E, and U? Explain your reasoning.

D) The battery is disconnected from the original capacitor, and its separation distance is doubled to $2d_o$. What are the new values of V, C, q, σ, E, and U? Explain your reasoning.

5. A capacitor is composed of two concentric spherical shells of radii a and b, respectively, that have equal and opposite charges as shown to the right. Just outside the surface of the inner shell, the electric field is directed radially outward and has magnitude E_o.



A) Using Gauss's law, express the charge $+Q$ on the inner shell as a function of E_o and a.

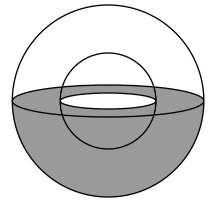
B) Write an expression for the electric field strength E between the shells as a function of E_o, a, and r.

C) Write an expression for the potential difference V between the shells as a function of E_o, a, and b.

D) Write an expression for the capacitance of the capacitor in terms of a and b.

E) Write an expression for the energy U stored in the capacitor as a function of E_o, a, and b.

F) A liquid of dielectric constant $\kappa = 3$ is poured into the bottom half of the capacitor, as shown to the right. By what factor has the capacitance of the capacitor increased? Explain your reasoning — a calculation is not required.



6. A parallel plate capacitor is made from two sheets of metal, each with an area of 1.0 square meter, separated by a sheet of plastic 1.0 millimeter (10^{-3} m) thick. The capacitance is measured to be 0.05 μFd.

A) What is the dielectric constant of the plastic?

The capacitor is charged by a battery to a potential of 30 volts.

B) What is the charge on each plate of the capacitor?

C) How much energy is stored in the fully charged capacitor?

After the capacitor is charged, it is disconnected from the battery, leaving the charged capacitor isolated. The plastic sheet is then removed from between the metal plates. The metal plates retain their original separation of 1.0 mm.

D) What is the new voltage across the plates?

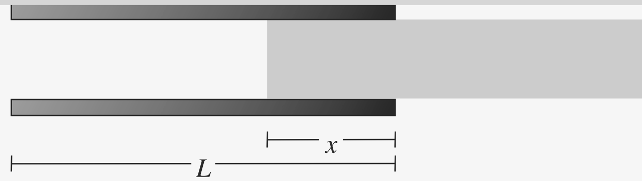
E) If there is now more energy stored in the capacitor, where did it come from? If there is less, what happened to it?

Now the original capacitor is fully charged to 30 volts, and left connected to the battery while the plastic sheet is removed.

F) What is the new charge on the plates?

G) If there is now more energy stored in the capacitor, where did it come from? If there is less, what happened to it?

7. A parallel plate capacitor has plates of length L and a capacitance C_o. The plates are charged to a potential difference V_o and then disconnected from the charging source. The space between the two plates is to be filled with a slab of dielectric constant κ.



A) The slab is inserted a distance x as shown to the right. As a function of the given variables and x, find

i) the capacitance C of the capacitor

ii) the potential difference V between the plates

iii) the energy U stored in the capacitor.

B) Once the slab is halfway into the capacitor, it is released. Assume that the slab has a mass m and neglect friction. Find the speed of the slab when it completely fills the capacitor.

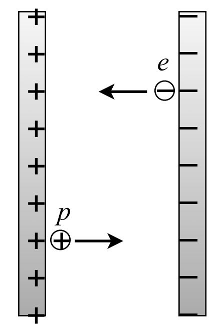
AP Physics C
Unit 11 Practice Test   Name ________________________

Part I
Multiple Choice

1. The two plates of a parallel-plate capacitor are a distance *d* apart and are mounted on insulating supports. A battery is connected across the capacitor to charge it and is then disconnected. The distance between the insulated plates is then increased to 2*d*. If fringing of the field is negligible, which of the following quantities is doubled?

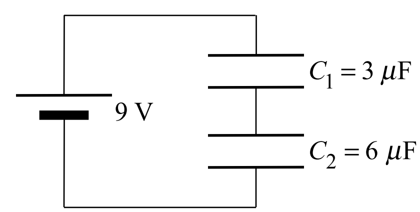
   A) The capacitance of the capacitor
   B) The total charge on the capacitor
   C) The surface density of the charge on the plates of the capacitor
   D) The energy stored in the capacitor
   E) The intensity of the electric field between the plates of the capacitor

2. A proton *p* and an electron *e* are released simultaneously on opposite sides of an evacuated area between large, charged parallel plates, as shown to the right. Each particle is accelerated toward the oppositely charged plate. The particles are far enough apart so that they do not affect each other. Which particle has the greater kinetic energy upon reaching the oppositely charged plate?



   A) The electron
   B) The proton
   C) Neither particle; both kinetic energies are the same.
   D) It cannot be determined without knowing the value of the potential difference between the plates.
   E) It cannot be determined without knowing the amount of charge on the plates.

3. Two capacitors initially uncharged are connected in series to a battery, as shown to the right. What is the charge on the top plate of C_1?



   A) $-81\ \mu C$
   B) $-18\ \mu C$
   C) $0\ \mu C$
   D) $+18\ \mu C$
   E) $+81\ \mu C$

4. A parallel-plate capacitor has a capacitance C_o. A second parallel-plate capacitor has plates with twice the area and twice the separation. The capacitance of the second capacitor is most nearly

   A) ¼ C_o
   B) ½ C_o
   C) C_o
   D) 2 C_o
   E) 4 C_o

For questions 5 and 6: Three 6-microfarad capacitors are connected in series with a 6 volt battery.

5. The equivalent capacitance of the set of capacitors is

   A) $0.5\ \mu F$
   B) $2\ \mu F$
   C) $3\ \mu F$
   D) $9\ \mu F$
   E) $18\ \mu F$

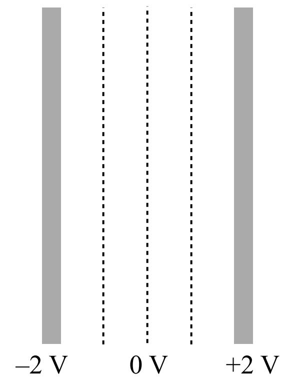
6. The energy stored in each capacitor is

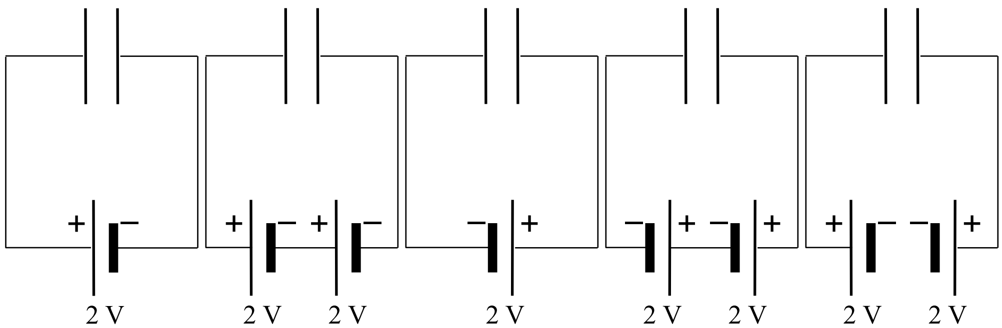
   A) $4\ \mu J$ B) $6\ \mu J$ C) $12\ \mu J$ D) $18\ \mu J$ E) $36\ \mu J$

7. A sheet of mica is inserted between the plates of an isolated charged parallel-plate capacitor. Which of the following statements is true?

   A) The capacitance decreases.
   B) The potential difference across the capacitor decreases.
   C) The energy of the capacitor does not change.
   D) The charge on the capacitor plates decreases
   E) The electric field between the capacitor plates increases.

8. A battery or batteries connected to two parallel plates produce the equipotential lines between the plates shown to the right. Which of the following configurations is most likely to produce these equipotential lines?



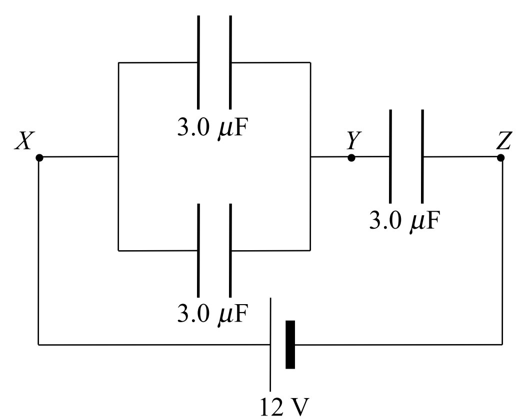


A) B) C) D) E)

9. A parallel-plate capacitor has charge $+Q$ on one plate and charge $-Q$ on the other. The plates, each of area A, are a distance d apart and are separated by a vacuum. A single proton of charge $+e$, released from rest at the surface of the positively charged plate, will arrive at the other plate with kinetic energy proportional to

   A) $\dfrac{edQ}{A}$
   B) $\dfrac{Q^2}{eAd}$
   C) $\dfrac{AeQ}{d}$
   D) $\dfrac{Q}{ed}$
   E) $\dfrac{eQ^2}{Ad}$

For questions 10 and 11*:* Three identical capacitors, each of capacitance 3.0 μF, are connected in a circuit with a 12 V battery as shown to the right.



10. The equivalent capacitance between points X and Z is

   A) 1.0 μF
   B) 2.0 μF
   C) 4.5 μF
   D) 6.0 μF
   E) 9.0 μF

11. The potential difference between points Y and Z is

   A) zero
   B) 3 V
   C) 4 V
   D) 8 V
   E) 9 V

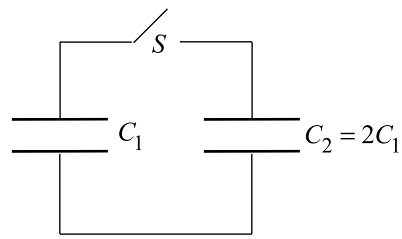
12. The units of capacitance are equivalent to

A) $\frac{\text{J}}{\text{C}}$ B) $\frac{\text{V}}{\text{C}}$ C) $\frac{\text{J}^2}{\text{C}}$ D) $\frac{\text{C}}{\text{J}}$ E) $\frac{\text{C}^2}{\text{J}}$

13. The capacitance of a spherical capacitor with inner radius a and outer radius b is proportional to

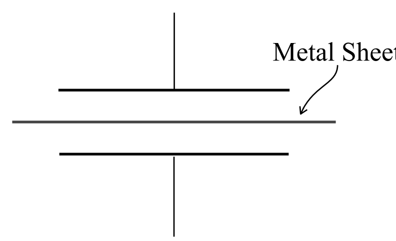
A) $\frac{a}{b}$
B) $b - a$
C) $b^2 - a^2$
D) $\frac{ab}{b-a}$
E) $\frac{ab}{b^2 - a^2}$

14. A capacitor of capacitance C_1 is charged and then connected to another initially uncharged capacitor of capacitance $C_2 = 2C_1$, as shown to the right, with the switch S in the open position. When S is closed and the system comes to equilibrium, which of the following is true of the charges on the capacitors and the potential differences across them?



Charge	Potential Difference
A) $Q_1 = \frac{1}{2}Q_2$	$V_1 = \frac{1}{2}V_2$
B) $Q_1 = \frac{1}{2}Q_2$	$V_1 = V_2$
C) $Q_1 = Q_2$	$V_1 = V_2$
D) $Q_1 = Q_2$	$V_1 = \frac{1}{2}V_2$
E) $Q_1 = 2Q_2$	$V_1 = V_2$

15. An air-gap capacitor originally has capacitance C. If a thin sheet of metal is placed halfway between the plates of the capacitor without touching either plate, as shown to the right, the effective capacitance is



A) $4C$
B) $2C$
C) C
D) $C/2$
E) $C/4$

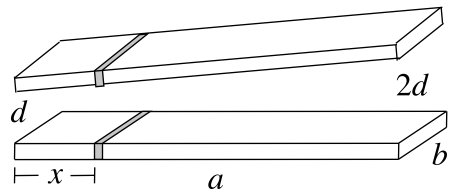
1.D 2.C 3.D 4.C 5.B 6.C 7.B 8.D 9.A 10.B 11D 12E 13D 14B 15C

Part II

Show your work

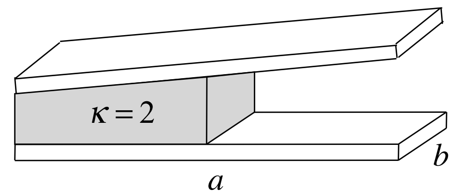
Credit depends on the quality and clarity of your explanations

1. A capacitor consists of two plates of length a and width b as shown to the right. The plates are not parallel, but are separated by a distance d on the left end, increasing linearly to $2d$ on the right end, with d very small compared to a and b. The shaded portion of each plate is at an arbitrary distance x from the left end, and has width dx.



A) What is the distance between the shaded portions of the plates?
B) What is the differential capacitance dC of the shaded portions?
C) Integrate to find the total capacitance of the capacitor.

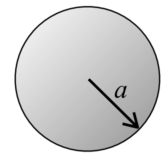
The left half of the capacitor is now filled with a dielectric with constant $\kappa = 2$ as shown to the right.



D) Determine the new capacitance of the capacitor.

2. An isolated conducting sphere of radius $a = 0.20$ m is at a potential of 2,000 V.

   A) Determine the charge Q_o on the sphere.

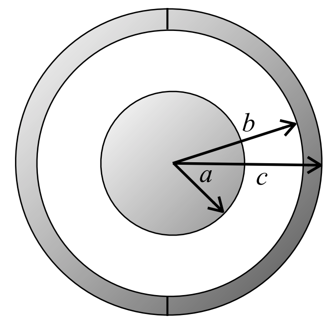


   The charged sphere is then concentrically surrounded by two uncharged conducting hemispheres of inner radius $b = 0.40$ m and outer radius $c = 0.50$ m, which are joined together as shown to the right, forming a spherical capacitor. A wire is connected from the outer sphere to ground, and then removed.

   B) Determine the magnitude of the electric field in the following regions as a function of the distance r from the center of the inner sphere.
      i) $r < a$
      ii) $a < r < b$
      iii) $b < r < c$
      iv) $r > c$

   C) Determine the magnitude of the potential difference between the sphere and the conducting shell.

   D) Determine the capacitance of the spherical capacitor.



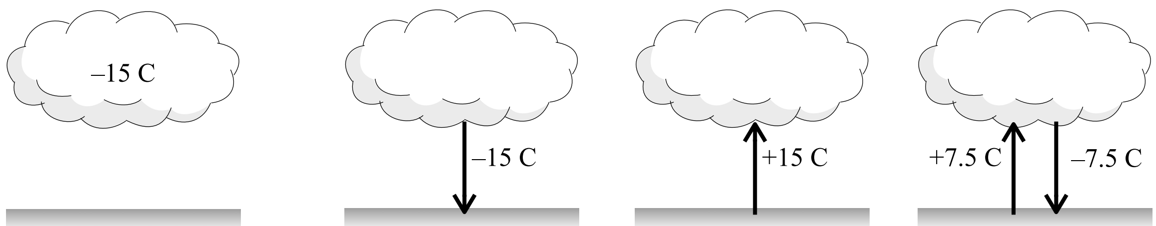
AP Physics C
Unit 12 Name____

Electric Current

We have defined *electric current i* as the *rate of flow of electric charge*: $i = \dfrac{dq}{dt}$. What is the SI unit of current?

This unit is defined as the *ampere* after André-Marie Ampère, a French physicist. We usually think of current as moving through a wire, but any net transfer of charge represents a current. For example, in a typical lightning strike, about 15 C of charge moves between a cloud and the earth in about 0.5 ms. How much current does this involve?

If the cloud starts with a net charge of –15 C, as shown below, determine the final charge on the cloud in the three scenarios shown.



Initial state

Final charge on cloud:

Final charge on cloud:

Final charge on cloud:

The discharge actually happens when the electric field $\vec{E}$ between the cloud and the earth builds up to about 3×10^6 V/m, at which point the electrons (pulled in the opposite direction of $\vec{E}$) are ripped from their nuclei (pulled in the same direction as $\vec{E}$). The free electrons travel in one direction and the ionized air molecules travel in the other. By convention, we agree that the direction of current flow is the direction of the *net flow of positive charge*, regardless of the sign or direction of the actual moving charge. According to this definition, what is the direction of the so-called "conventional current" in each of the scenarios above?

Current Density

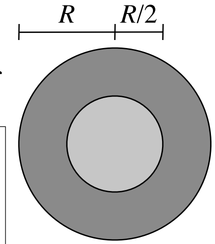
The very large currents involved in lightning are all the more spectacular because they are confined to a very narrow channel; the typical lightning bolt has a radius of about 1 cm. We define the *current density J* as the current per cross-sectional area:

What are the SI units of current density? What is the current density of a typical lightning bolt?

Power lines typically carry about 250 A along a cable with about the same radius. What is the current density in a power line?

We can imagine a situation in which the current density is not uniform; for example, the current density in the center of a lightning bolt might be greater or smaller than the current density toward the edges. A more precise definition of current density allows for this: At some point within the current flow, consider a small area *dA* with a small current *di* flowing through it. Then at this point, $J = \dfrac{di}{dA}$. To find the total current, we would have to integrate over the whole cross-sectional area.

Example



To the right is a cross section of a lightning bolt of radius R.

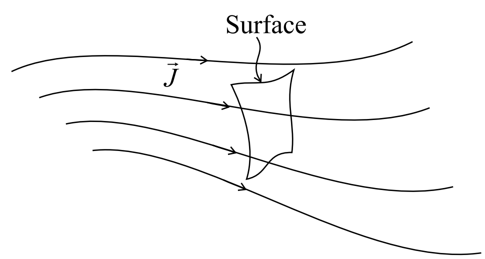
A) If the current density J is uniform, what fraction of the total current is carried by the outer part of the bolt between $R/2$ and R?

B) Instead, let the current density vary with distance from the center of the conductor as $J = ar^2$, where a is a constant.

i) Determine the total current in terms of a and R.

ii) What fraction of the total current is carried by the outer part of the bolt?

We can be even more precise about our definition of current density by defining its direction: It is the direction in which net positive charge flows at that point, which is the same as the direction of the electric field $\vec{E}$ at that point. If current density has magnitude and direction, we can represent it as a *vector*, and if it has a value at every point in the current flow, it is a *vector field*.

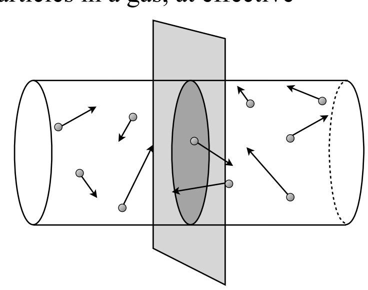


Consider an arbitrary surface in a region of space in which a nonuniform current density exists, as shown to the right. At different points on the surface, the current density may have different magnitudes, and be at different angles to the surface. How would you use the concept of *flux* to calculate the current that passes through the surface?

Current in a Wire

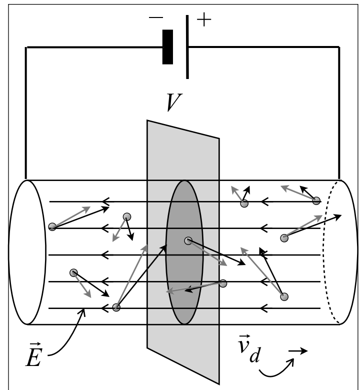
Even before the discovery of electrons, experiments showed that the charge carriers in a conductor were negative. We now understand that in a typical conductor carrying no current, there are electrons (usually one per molecule) that are not attached to a particular molecule, but move randomly between molecules, much like the motion of particles in a gas, at effective speeds on the order of 10^6 m/s.

Consider an arbitrary cross section of the conductor, depicted on the right as the circle obtained by passing a plane through a cylindrical wire. In a given time, many electrons will pass through this cross section, but there is no *net* flow of charge because on the average, the same number of electrons cross to the right as to the left. A very sensitive ammeter, however, would flutter back and forth about a zero reading, with small currents in either direction based on the statistics of the random motion. This “noise” can be a problem for researchers measuring very small currents.



Wire with no current

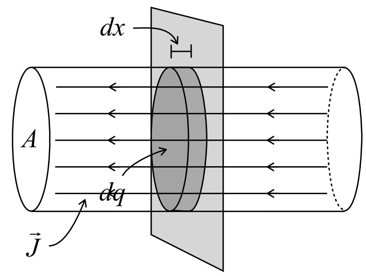
Drift Velocity



Wire with current

If the ends of the conductor are connected to the terminals of a battery, there will be a potential difference between the ends, resulting in an electric field inside the wire. This slightly shifts the motions of the electrons, resulting in a net flow through the cross section. This small additional velocity, called the *drift velocity* $\vec{v}_d$, is exaggerated in the drawing.

We have seen before that $\vec{E}$ is zero inside a conductor at electrostatic equilibrium, but this wire is not at equilibrium. Choose a Gaussian surface that lies entirely within the segment of wire, and draw it on the diagram. Will there be a nonzero flux for your surface? Explain.



Assume that there are *n* molecules per unit volume in the conductor, and that each molecule contributes one conduction electron with the elementary charge *e*. We wish to determine the drift velocity in this conductor if the current density is *J*.

The drawing to the right shows a cylinder of charge of length *dx* passing through a cross sectional area of the wire in time *dt*.

A) How many molecules are in the cylinder?

B) How much charge *dq* is in the cylinder?

C) What is the current *i* in the conductor?

D) What is the current density *J* in the conductor?

E) Express the drift velocity in terms of *J*, *n* and *e*:

Example

The thickness of wire is described by its *gauge*; the smaller the gauge the thicker the wire. A 20-gauge copper wire carries a maximum safe current of 1.5 amperes. Assume that copper has on average one conduction electron per atom, and use the information to the right:

- 20-gauge wire has a diameter of 0.032 inches.
- There are 2.54 cm in 1 inch.
- The mass density of copper is 8.96 grams/cm^3
- The molar mass of copper is 63.5.
- Avogadro's number N_A is 6.02×10^{23}.
- The elementary charge *e* is 1.6×10^{-19} C
- The mass of an electron is 9.11×10^{-31} kg

A) Calculate the maximum current density *J* in the wire.

B) Calculate the number *n* of copper atoms per cubic meter.

C) Determine the drift velocity of the conduction electrons at the maximum current density.

In a 12 V automotive electrical system, a 20-gauge wire is used to supply 0.5 A of current to the taillight.

D) How long does it take for a conduction electron to make the round trip from the battery to the taillight and back, a distance of about 6 m?

E) Express this time in the most appropriate of the following units: nanoseconds, microseconds, milliseconds, seconds, minutes, hours, days:

F) Assume that the electric field responsible for the current in the taillight wire is uniform throughout the 6 m round trip. (We'll make a more realistic calculation on the next page.) What is the strength of this electric field?

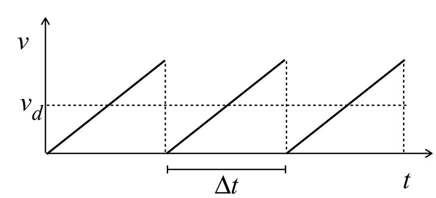
G) Determine the acceleration of the conduction electrons in the taillight wire due to this electric field.

H) How long would it take for a conduction electron, starting from rest, to accelerate to the drift velocity?

I) Express this time in the most appropriate of the following units: nanoseconds, microseconds, milliseconds, seconds, minutes, hours, days:

Resistivity

Clearly something must be slowing the conduction electrons down; as they are pulled along by the electric field, they collide with the molecules in the conducting material. Let's consider a simple model of this motion: A conduction electron starts at rest at one molecule, is accelerated by the electric field until it hits the next molecule, where it is brought to rest by the collision. Its kinetic energy is transferred to the molecule in the form of thermal energy, and the electron begins its next step of the journey.



The graph to the right represents the speed of the electron along its path. The time Δt represents the average time between collisions. Call the electric field responsible for the current E, the charge of the electron e, and its mass m_e. Assume that there are n conduction electrons per unit volume in the conducting material.

A) Determine the drift velocity v_d in terms of these quantities.

B) Express the electric field in terms of the current density J and these quantities.

The relationship between the electric field and the current density depends on some factors that are constant, such as the properties of the electron, and others that are different for different conducting materials, such as the number of conduction electrons per unit volume and the average time between collisions. These factors are lumped together into one value called the *resistivity* of the conducting material, symbolized by ρ (rho):

$$\vec{E} = \rho \vec{J}$$

For a given electric field, a material with a high resistivity will have a low current density, and vice versa. What are the SI units of resistivity? Express these units in terms of volts V, amperes A, and meters m:

Example

The resistivity of copper has the numeric value 1.69×10^{-8} in these units. Assume that the taillight wire in the previous example is made of copper.

A) Use the relationship above to determine the electric field responsible for the current in the taillight wire.

B) Determine the acceleration of the conduction electrons in the taillight wire due to this electric field.

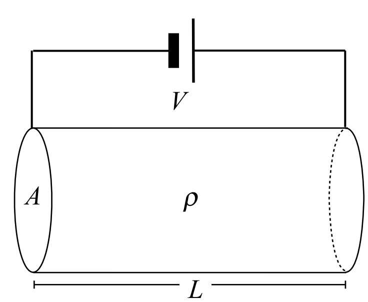
C) How long would it take for a conduction electron, starting from rest, to accelerate to the drift velocity?

D) What is the potential difference between the ends of the 6 m of wire corresponding to this electric field?

Clearly most of the potential difference of 12 V occurs somewhere besides the wire. In fact most of the potential difference, and therefore the energy given off, takes place in the filament of the taillight bulb. The filament is designed to convert the energy of the electrons into heat, and it will glow brightly when it's very hot.

Resistance

The drawing to the right represents a segment of wire of cross-sectional area *A* and length *L*, made of a material with resistivity ρ. A battery supplies a potential difference *V* between the ends of the wire.



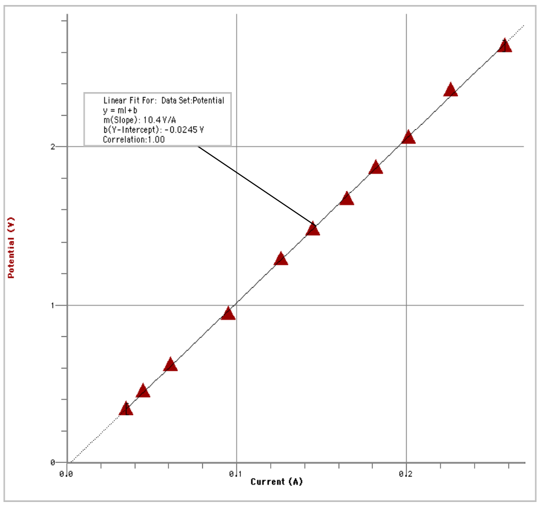
A) What is the electric field within the wire?

B) Find the ratio of the potential difference *V* to the current *i* in the wire.

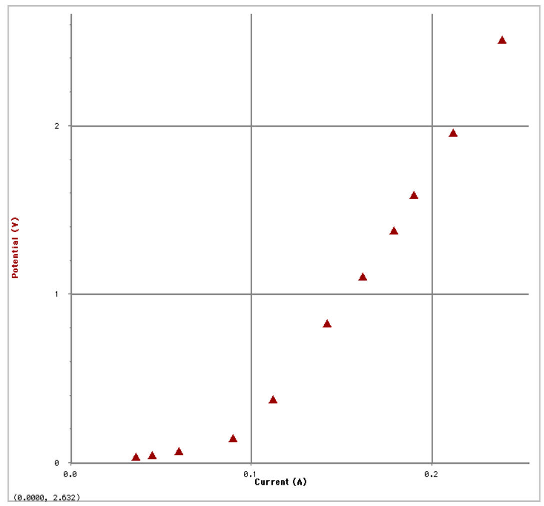
This ratio is called the *resistance R* of this particular segment of wire. Note that resistance is a property of particular items, whereas resistivity is a general property of materials of which the items are made.

What is the SI unit of resistance? This unit is called the *Ohm*, after the German physicist Georg Ohm, and symbolized by Ω (capital omega).

The resistivity of some materials may depend on factors such as temperature, and if so the resistance of items made of that material will not be constant; ie., the ratio of potential to current will vary. The graphs below show the result of applying varying voltages to a *resistor*, which has a constant resistance, and a light bulb, whose filament has a resistance that depends on temperature. As more current is sent through the filament of the bulb, it heats up, so the resistance changes. Items such as the resistor are said to obey *Ohm's Law*, and are called *ohmic*.



Ohmic Resistor



Non-ohmic Light Bulb

Examples

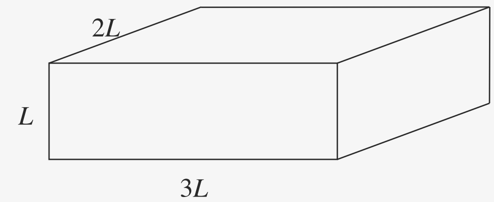
1. A certain material has a resistivity ρ and charge-carriers per volume *n*. A potential difference *V* is applied to a wire of length *L* made of this material.
   A) Express the drift velocity of the electrons in the wire in terms of these quantities and constants.

   B) Check your expression for dimensional consistency, using SI units.

2. A researcher has a mass *m* of a conducting material whose resistivity is ρ, and whose mass density is δ. She wishes to form a wire whose resistance is *R*.
   A) Derive a formula for the length *L* of the wire in terms of the given variables.

B) Check the equation derived in A) for dimensional consistency, using SI units.

3. The figure to the right shows a rectangular solid conductor of resistivity ρ and edge lengths L, $2L$ and $3L$. A certain potential difference V is to be applied between pairs of opposite faces of the conductor (left-right, top-bottom, front-back). In each case find the given quantities.



A) Left-right:

i) Resistance R

ii) Current i

iii) Current density J

B) Top-Bottom:

i) Resistance R

ii) Current i

iii) Current density J

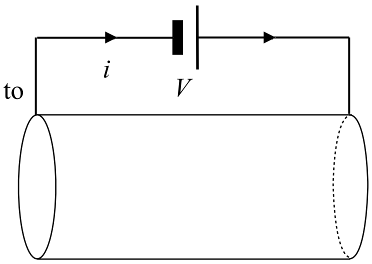
C) Front-back:

i) Resistance R

ii) Current i

iii) Current density J

Energy Transfers and Ohm's Law



The collisions of the conduction electrons transfer energy gained in the battery to the molecules of the resistor, heating it up. We'll consider the wires in the diagram to the right to have no resistance, so that all the energy transfer takes place in the resistor.

If a small charge *dq* moves through the battery in a time *dt*, how much energy *dU* is gained from the battery? Express the *rate* at which this energy is gained in terms of the current *i* and the potential difference *V*: Rate of transfer of energy is *power*, as we have seen in mechanics. Power is measured in *watts*, where 1 watt = 1 J/s. From your expression above, what is another way to express 1 watt?

The expression above for power expresses both the rate of energy gain in the battery and, by conservation of energy, the rate at which it is used in the circuit. If the external circuit is a resistor that obeys Ohm's Law, we can express the power dissipated (transferred out of the circuit) in terms of its resistance *R* and one other quantity: Express the power of an ohmic resistor in terms of: *i* and *R*: *V* and *R*:

Example

A 1200 watt heater is designed to operate at 120 V. The heating element is made of 22 gauge nichrome wire, whose resistivity is $1.5\times10^{-6}\ \Omega\text{m}$. 22 gauge wire has a radius of 0.321 mm.

A) Determine the resistance of the heating element.

B) Determine the length of nichrome wire in the heating element.

AP Physics C
Unit 12 Practice Quiz Name ____________________________________

Part I
Multiple Choice

1. A car battery is rated at 80 A•h. An ampere•hour is a unit of

   A) power
   B) energy
   C) current
   D) charge
   E) force

2. Two wires made of different materials have the same uniform current density. They carry the same current only if

   A) their lengths are the same
   B) their cross-sectional areas are the same
   C) both their lengths and cross-sectional areas are the same
   D) the potential difference across them are the same
   E) the electric fields in them are the same

3. The current is zero in a copper conductor when no potential difference is applied because

   A) the electrons are not moving
   B) the electrons are not moving fast enough
   C) for every electron with a given velocity there is another with a velocity of equal magnitude in the opposite direction
   D) equal numbers of electrons and protons are moving together
   E) otherwise Ohm's Law would not be valid

4. A certain piece of material carries a current of 2 amperes when the potential difference is 1 V, and a current of 5 amperes when the potential difference is 2 V. This material

   A) has a resistance of 2.5 Ω at 2 V
   B) has a resistance of 2 Ω at 1 V
   C) has a resistance of 0.4 Ω at 2 V
   D) does not have resistance
   E) obeys Ohm's Law

For questions 5 - 7: Brianna kept her 75 watt, 120 volt desk lamp turned on from 4 pm until 2 am. HECO charges 35 cents per kW•h.

5. How many coulombs of charge would have passed through the lamp?

   A) 190
   B) 4,500
   C) 9,000
   D) 11,250
   E) 22,500

6. About how much did it cost Brianna to keep her light on?

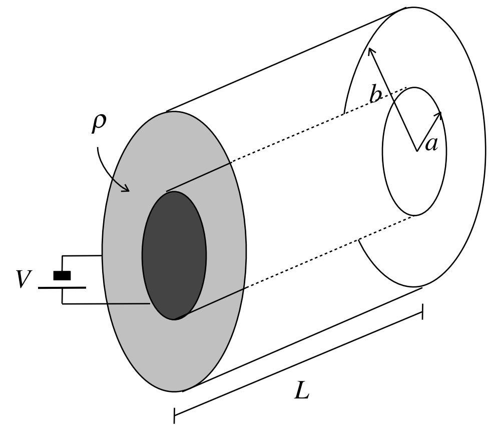
   A) a penny
   B) a nickel
   C) a dime
   D) a quarter
   E) a dollar

7. When Brianna bought the 75 watt light bulb for his desk lamp, she knew that

   A) no matter how she used the bulb, the power would be 75 watts.
   B) the bulb was filled with 75 watts at the factory.
   C) the actual power dissipated will be much greater than 75 watts since most of the power appears as heat
   D) the bulb is expected to burn out after she uses up its 75 watts.
   E) none of these

1. D 2. B 3. C 4. C 5. E 6. D 7. E

Part II
Show your work
Credit depends on the quality and clarity of your explanations



1. A rod of negligible resistance, length L and radius a is surrounded by a cylindrical electrode of inner radius b, also of length L and negligible resistance. The material between the rod and the electrode has a resistivity ρ. A battery establishes a potential difference V between the rod and the electrode, so that a radial current flows from the rod outward to the cylinder.

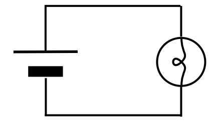
   A) Consider a cylindrical shell of radius r ($a < r < b$), thickness dr, and length L. Express the differential resistance dR of this shell for radial current, in terms of the given variables.
   B) Integrate to find the total resistance between the rod and the electrode.
   C) Express the current density as a function of r, the distance from the axis of the rod, where $a < r < b$.

AP Physics C
Unit 13 Name________________________________

Electric Circuits

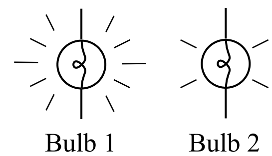
In Unit 12 we examined current and resistance in conducting materials, and developed a simple model for the transfer of energy in a circuit. Charges flowing through a battery gain energy (from a chemical reaction), and this energy is given off by the charges in the outside circuit, often in the form of thermal energy.

We'll simplify our discussion of a battery-bulb circuit by assuming that the wires have negligible resistance, and the filament of the bulb behaves as an ohmic resistor (ie., obeys Ohm's Law). Charges gain energy in the battery and dissipate the energy in the bulb. The brightness of the bulb is an indication of the rate at which it converts energy to thermal energy (and light).



If a current i is flowing through the bulb in the circuit above, and the potential difference across the bulb is V, what is the rate at which it converts energy? What is another word for this rate?

Bulbs 1 and 2 are otherwise identical, but bulb 1 is brighter than bulb 2.

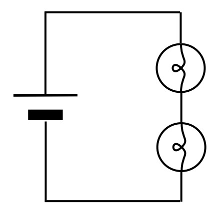


A) Is the current in bulb 1 necessarily greater than the current in bulb 2? Explain.

B) Is the potential difference across bulb 1 necessarily greater than the potential difference across bulb 2? Explain.

Bulbs in Series

Two identical bulbs are connected in series in the circuit to the right, with the same battery as the single bulb circuit above.



A) Will the current in the top bulb be *greater than*, *less than*, or *the same as* the current in the bottom bulb? Explain in terms of the definition of current.

B) Will the potential difference across each bulb be *greater than*, *less than*, or *the same as* the potential difference across the battery? Explain in terms of the energy exchanges in the circuit.

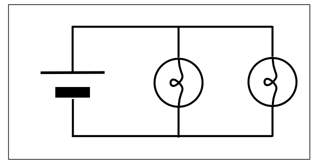
C) Will the bulbs in this circuit be *brighter*, *dimmer*, or *the same brightness* as the bulb in the single-bulb circuit? Explain in terms of the energy exchanges in the circuit.

D) Will the current in the battery in this circuit be *greater than*, *less than*, or *the same as* the current in the battery in the single bulb circuit? Explain.

G) Is the resistance of this combination of two bulbs *greater than*, *less than*, or *the same as* the resistance of a single bulb? Explain using the definition of resistance.

H) In general would adding additional bulbs in series *increase*, *decrease* or *not change* the total resistance of the combination? Explain in terms of Ohm's Law and how the potential difference and current for each bulb is affected.

Bulbs in Parallel



Two identical bulbs are connected in parallel in the circuit to the right, with the same battery as the single bulb and series circuits above.

A) On the diagram to the right, sketch and number all possible paths a charge could take in going from the positive terminal around the circuit to the negative terminal of the battery.

B) For each of these paths, will the energy transferred to a bulb by a given charge be *greater than*, *less than*, or *the same as* the energy gained in the battery by that charge? Explain.

C) Will the potential difference across the left bulb be *greater than*, *less than*, or *the same as* the potential difference across the right bulb? Explain in terms of the definition of potential difference.

D) Will the potential difference across each bulb be *greater than*, *less than*, or *the same as* the potential difference across the battery? Explain in terms of the energy exchanges in the circuit.

E) Will the bulbs in this circuit be *brighter*, *dimmer*, or *the same brightness* as the bulb in the single-bulb circuit? Explain in terms of the energy exchanges in the circuit.

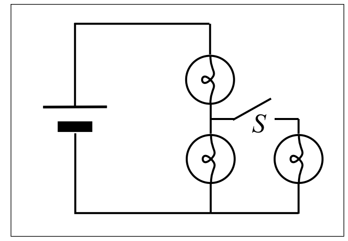
F) Will the current in the battery in this circuit be *greater than*, *less than*, or *the same as* the current in the battery in the single bulb circuit? Explain in terms of the definition of current.

G) Is the resistance of this combination of two bulbs *greater than*, *less than*, or *the same as* the resistance of a single bulb? Explain using the definition of resistance.

H) In general would adding additional bulbs in parallel *increase*, *decrease* or *not change* the total resistance of the combination? Explain in terms of Ohm's Law and how the potential difference and current for each bulb is affected.

Download the PhET Circuit Construction Kit from the web site. Launch the simulator, click the "Small" button under "Size" and construct the three circuits we have discussed above, so that all three appear in the workspace. Verify that your predictions about the brightness of the bulbs in each circuit is correct. If not, resolve any inconsistencies.

Series and Parallel Combinations



When the switch *S* is closed in the circuit to the right, a third bulb is connected in parallel with the bottom bulb in the original series circuit.

A) On the diagram, sketch and number all possible paths a charge could take in going from the positive terminal around the circuit to the negative terminal of the battery once the switch is closed.

B) Does the current change along a given path, or remain the same? Explain in terms of the definition of current.

C) How does the current in the top bulb compare to the current in the bottom left bulb?

D) How does the potential difference across the top bulb compare to the potential difference across the bottom left bulb? Explain using Ohm's Law.

E) How, if at all, does the potential difference across the top bulb *change* when the switch is closed? Explain in terms of the fraction of the battery's potential across the top bulb before and after the switch is closed.

F) How, if at all, does the current in the top bulb *change* when the switch is closed? Explain using Ohm's Law.

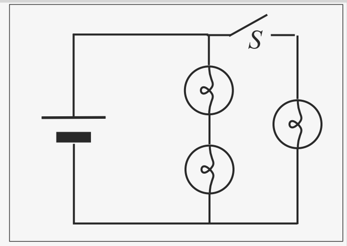
G) How, if at all, does the current in the battery *change* when the switch is closed? Explain how you know.

H) According to Ohm's Law, is the resistance of the circuit with the switch closed *greater than*, *less than*, or *the same as* the resistance of the circuit with the switch open? Explain.

I) A student makes the following statement: "The lower bulb will dim when the switch is closed, because it now only gets half the current. But the brightness of the top bulb won't change because it still gets all the current." What's wrong with this statement?

J) Use the PhET simulator to construct the circuit. Verify that your predictions about the brightness of the bulbs before and after the switch is closed is correct. If not, resolve any inconsistencies.

When the switch S is closed in the circuit to the right, a third bulb is connected in parallel with both bulbs in the original series circuit.



A) On the diagram, sketch and number all possible paths a charge could take in going from the positive terminal around the circuit to the negative terminal of the battery once the switch is closed.

B) How, if at all, does the potential difference across the top bulb *change* when the switch is closed? Explain in terms of energy transfers along different paths in the circuit.

C) How, if at all, does the current in the top bulb *change* when the switch is closed? Explain using Ohm's Law.

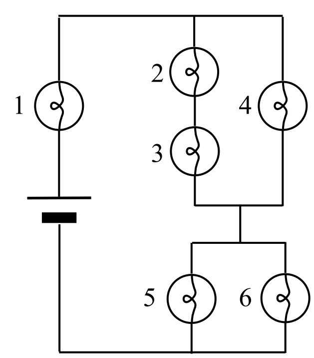
D) How does the potential difference across the third bulb compare to the potential difference across the battery once the switch is closed? Explain in terms of energy transfers along different paths in the circuit.

E) How does the current in the battery *change* when the switch is closed? Explain how you know.

F) According to Ohm's Law, is the resistance of the circuit with the switch closed *greater than*, *less than*, or *the same as* the resistance of the circuit with the switch open? Explain.

G) Use the PhET simulator to construct the circuit. Verify that your predictions about the brightness of the bulbs before and after the switch is closed is correct. If not, resolve any inconsistencies.

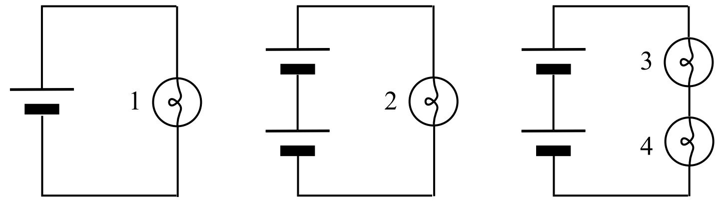
Six identical bulbs are connected as shown in the circuit to the right. Rank the brightness of the bulbs, and explain your reasoning.



Batteries in Series

Consider the three circuits to the right, made with identical batteries and bulbs.

A) Predict the ranking of the brightness of the bulbs from brightest to dimmest. If any bulbs have equal brightness, state so explicitly.



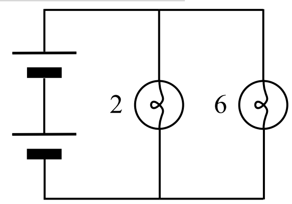
Circuit I Circuit II Circuit III

B) Explain your ranking in terms of the energy exchanges in each circuit.

C) Use the PhET simulator to construct the circuits so that all three appear in the workspace. Verify that your predictions about the brightness of the bulbs in each circuit is correct. If not, resolve any inconsistencies.

Circuit IV is constructed from Circuit II by adding a bulb in parallel to bulb 2.

D) Will the brightness of bulb 2 *increase*, *decrease*, or *remain the same*? Explain.



Circuit IV

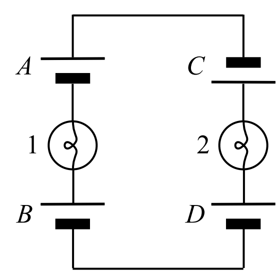
E) Use the PhET simulator to construct the circuit by adding another bulb to Circuit II. Verify that your predictions about the brightness of bulb 2 is correct. If not, resolve any inconsistencies.

In the early days of the study of batteries, the term used to describe the energy per unit charge given to the charges flowing through a battery was *electromotive force*, or *emf* $\mathcal{E}$. We still use the abbreviation even though energy per unit charge is not a force. Make a general statement about the effective emf of multiple batteries when connected in series.

We can think of the direction of a battery's emf as pointing out the positive terminal. The chemical reaction that gives energy to charges passing through a battery may be reversible. In that case, charges sent through the battery in the opposite direction of its emf will *give* energy *to* the battery; we say that the battery is *rechargeable* and that it is being *charged*. It would be better to say that it is being *energized* because it is gaining energy, not charge. A battery giving energy to a circuit is said to be *discharging*. We will assume in what follows that our batteries are rechargeable.

Consider the circuit to the right, with four batteries and two bulbs. Note the orientation of the batteries.

A) How many distinct paths are there for a charge that leaves the positive terminal of battery A and goes completely around the circuit?



Circuit V

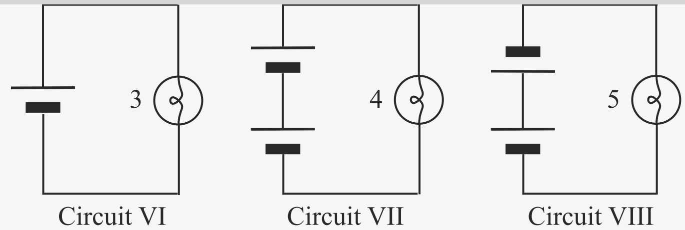
B) Rank the brightness of the two bulbs and explain your reasoning.

C) In what direction, if any, will current flow in the circuit? Explain your reasoning in terms of energy transfers in the circuit.

D) Which battery(ies) is (are) being charged? Explain.

Consider circuits VI - VIII to the right.

E) Rank the brightness of bulbs 1 - 5 in circuits V - VIII.



F) Explain the reasoning for your rankings above.

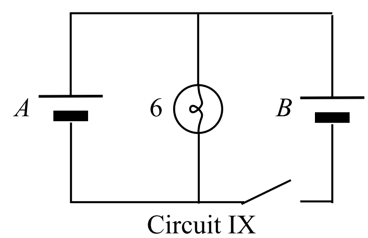
G) Use the PhET simulator to construct the circuits. Verify that your predictions about the relative brightness of the bulbs is correct. If not, resolve any inconsistencies.

Modify your general statement on the previous page about the effective emf of batteries in series to account for the direction of each battery's emf.

Multiple Loop Circuits

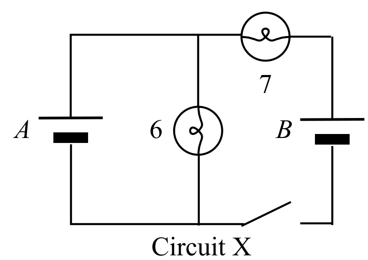
Consider Circuit IX to the right.

A) Predict whether the brightness of bulb 6 will *increase*, *decrease*, or *remain the same* when the switch is closed. Explain your reasoning.



Circuit X is formed by adding bulb 7 as shown.

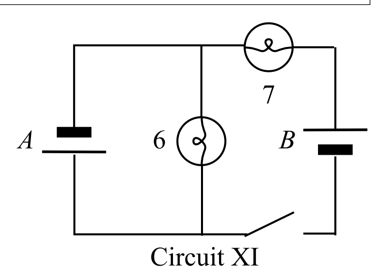
B) Predict whether the brightness of bulb 6 will *increase*, *decrease*, or *remain the same* when the switch is closed. Explain your reasoning.



C) How will the brightness of bulb 7 compare to the brightness of bulb 6 once the switch is closed? Explain your reasoning.

Circuit XI is formed by reversing battery *A* as shown.

D) Predict whether the brightness of bulb 6 will *increase*, *decrease*, or *remain the same* when the switch is closed. Explain your reasoning.



E) How will the brightness of bulb 7 compare to the brightness of bulb 6 once the switch is closed? Explain your reasoning.

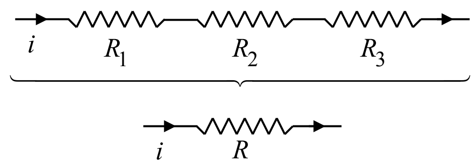
F) Use the PhET simulator to construct these circuits and verify that your predictions are correct. If not, resolve any inconsistencies. **Complete this PhET activity, don't skip!**

Quantitative Analysis of Circuits

We will now develop tools based on what we have learned so far that will enable us to calculate the behavior of circuits. Instead of bulbs, we will consider the more generic *resistors*, whose symbol is shown to the right, which may have various resistances. We will see how they combine with batteries in various ways.


Resistor

Three resistors are connected in series, as shown to the right. A current i is sent through the combination. We wish to find the resistance R of a single resistor with the same resistance as the series combination.

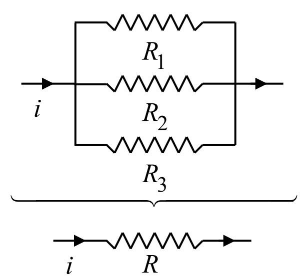


A) What is the potential difference V across the combination of resistors in terms of the potential differences V_1, V_2, and V_3 across each resistor?

B) What is the current i through the combination of resistors in terms of the currents i_1, i_2, and i_3 through each resistor?

C) Use Ohm's Law to express the total resistance R in terms of the resistances R_1, R_2, and R_3:

Three resistors are connected in parallel, as shown to the right. A current i is sent through the combination. We wish to find the resistance R of a single resistor with the same resistance as the parallel combination.



A) What is the potential difference V across the combination of resistors in terms of the potential differences V_1, V_2, and V_3 across each resistor?

B) What is the current i through the combination of resistors in terms of the currents i_1, i_2, and i_3 through each resistor?

C) Use Ohm's Law to express the total resistance R in terms of the resistances R_1, R_2, and R_3:

We saw in Unit 12 that charges moving through a conductor encounter obstacles, which give rise to resistivity of materials and resistance of circuit elements. This is also true for batteries themselves; they have resistance called *internal resistance*. Current generates heat no matter which direction it flows through a battery.

A) Select "Reset All" on the PhET simulator to start with a blank workspace. Drag in a battery and click the Voltmeter checkbox. Position the probes of the voltmeter on the terminals of the battery and read the voltage:

B) Construct a complete circuit with this battery and one resistor. Measure the voltage of the battery while it is discharging:

C) Control-click on the battery and select "Change Internal Resistance." Type in 2.0 Ω. Measure the voltage of the battery while it is discharging:

D) Disconnect one of the wires in the circuit to stop the current. Measure the voltage of the battery:

E) Explain why the terminal voltage of the battery when it is discharging is less than when it is not.

Add two more batteries to the circuit in series so that there are three in series with the resistor. Change the internal resistance of these batteries to 2 Ω. Control-click on one of the batteries and select "Reverse" so that this battery is being charged.

F) Measure the terminal voltage of the charging battery:

G) Measure the terminal voltage of the discharging batteries:

H) Explain why the terminal voltage of the charging battery is greater than its emf.

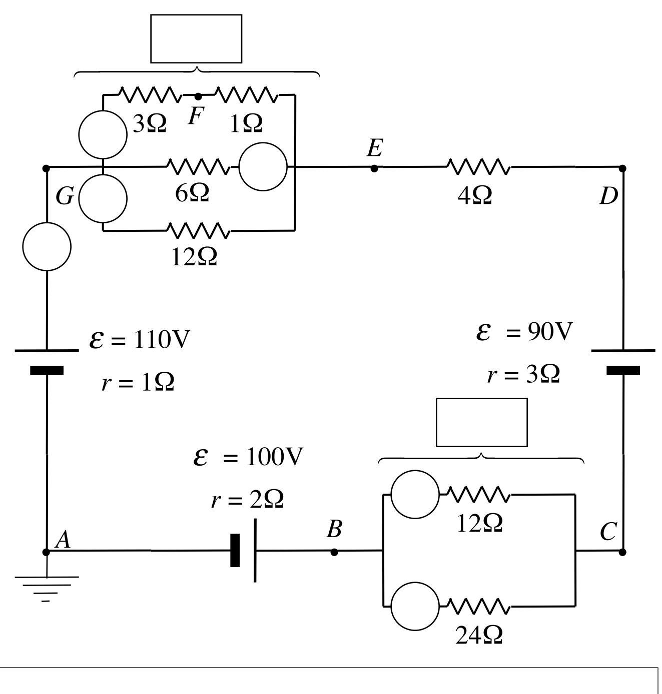
I) Write expressions for the terminal voltage of a battery with emf $\mathcal{E}$, internal resistance r, carrying a current i under the following conditions:

i) The battery is discharging:

ii) The battery is charging:

Simple Circuits

Circuits that are equivalent to a single emf and a single resistance are called *simple circuits*, no matter how complicated they may look. The circuit to the right is a simple circuit.



A) Determine the equivalent emf of the circuit.

B) Determine the equivalent resistance of the parallel section on the top. Write this value in the box.

C) Determine the equivalent resistance of the parallel section on the bottom. Write this value in the box.

D) Determine the equivalent resistance of the circuit, including the internal resistances of the batteries.

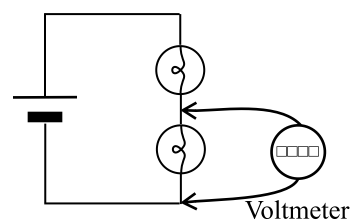
E) Determine the maximum current in the circuit, and write it in the appropriate circle.

F) Determine the currents in the remainder of the circuit, and write them in the appropriate circles. Show your work.

G) The symbol at point *A* means that point *A* is *ground*; ie., it is arbitrarily set to zero potential. Determine the potentials of the remaining lettered points relative to *A*. Show your work.

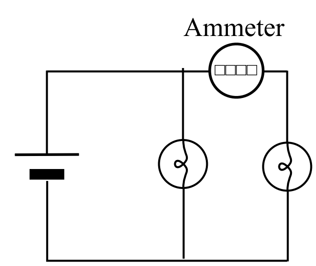
Ammeters and Voltmeters

In order to measure the potential difference between two points in a circuit, we simply place the red probe at the point with the higher potential and the black probe at the point with the lower potential. The meter reads the potential between the points. The voltmeter in the diagram to the right is measuring the potential difference across the bottom bulb.



A) Should the voltmeter itself have a *high resistance* or a *low resistance* in order not to affect the potential difference it is measuring? Explain.

In order to measure the current at a point in a circuit, we place an ammeter into the circuit in such a way that all of the current to be measured passes through it. The ammeter in the diagram to the right is measuring the current in the bulb on the right.



B) Should the ammeter itself have a *high resistance* or a *low resistance* in order not to affect the current it is measuring? Explain.

The same digital display can be used by a manufacturer to construct either of these instruments. In fact, most meters are "multimeters" which are capable of measuring either voltage or current by flipping a switch. The switch engages the proper circuitry with the display so that the proper quantity is measured.

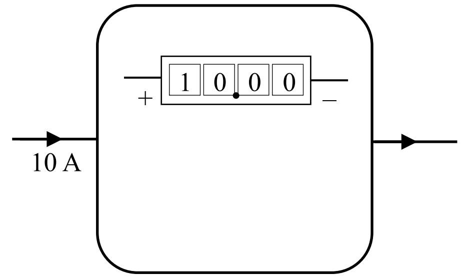
Example

Your company has bought thousands of digital displays with the following specifications:

- Each display has two terminals labeled "+" and "–"
- The internal resistance of the display is 10 Ω.
- The reading on the display is proportional to the current sent through it.
- When a current of 10 mA is sent through the display, it reads "10.00". More current damages the display.

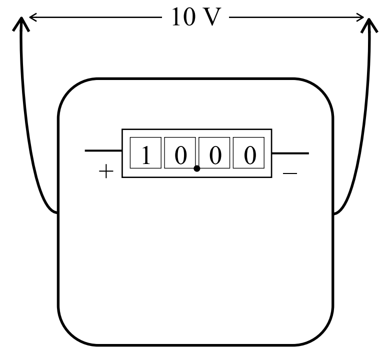
Your job is to design a multimeter with two switch settings: "10 A" and "10 V" so that:

A) When the switch is set to "10 A" and 10 amperes of current is sent through the multimeter, the display reads "10.00". Calculate the resistance of the single resistor you need to connect to the display, and show how it is connected in the diagram. This resistor is called a "shunt."



Ammeter

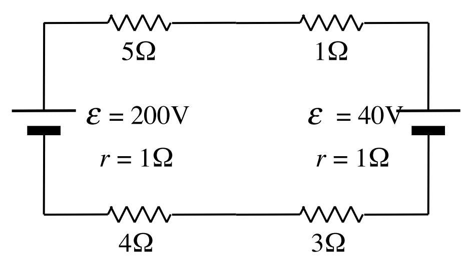
B) When the switch is set to "10 V" and the terminals of the multimeter are connected to a potential difference of 10 V, the display reads "10.00". Calculate the resistance of the single resistor you need to connect to the display, and show how it is connected in the diagram. This resistor is called a "multiplier."



Voltmeter

Multiloop Circuits and Kirchhoff's Rules

Consider the simple circuit to the right.

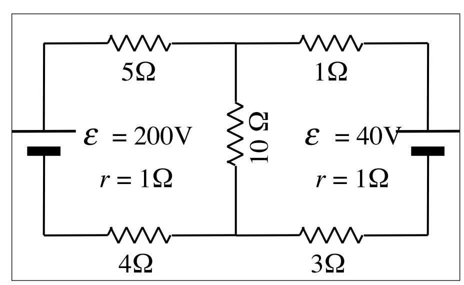


A) Determine the equivalent emf of the circuit:

B) Determine the equivalent resistance:

C) Determine the current in the circuit:

We now add a 10 Ω resistor as shown to the right. It is impossible to resolve the circuit into a single equivalent emf and a single resistance. Such a circuit is analyzed by two tools called *Kirchhoff's Rules* after Gustav Kirchhoff, a German physicist.



1. (*The Junction Rule*) The net current flowing into a junction is equal to the net current flowing out of the junction.
2. (*The Loop Rule*) The net change in potential around any closed loop in the circuit is zero.

D) In the circuit to the right, how many different currents are there?

E) Although you don't necessarily know in advance the direction of the currents, choose a direction for each current and label them i_1, i_2, etc. on the diagram.

F) Write the equation that results from applying the Junction Rule to one of the junctions:

G) Choose two loops that you can follow *in the direction of your chosen current*. Write the two equations that result from applying the Loop Rule to these loops.

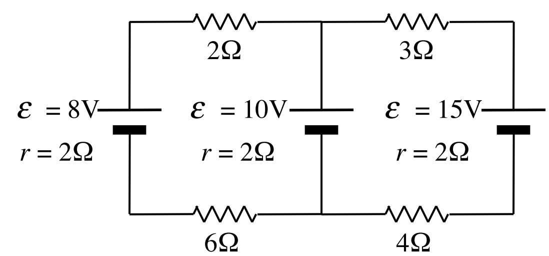
H) Solve these equations to find the currents in each part of the circuit. If any result is negative, it just means that the current goes in the direction opposite the one you chose.

Consider the multiloop circuit to the right.

A) Label each current on the diagram.

B) Write the Junction Rule equation:

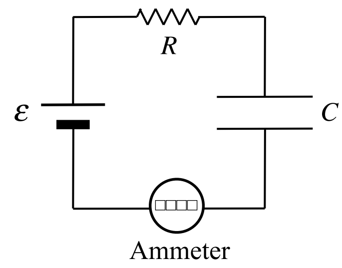
C) Write the Loop Rule equations:



D) Solve to find the currents.

RC Circuits

In the Capacitor Lab you used a circuit like the one to the right to observe the current while the capacitor was being charged. Such a circuit is called a *Simple RC circuit*.



Imagine that we start charging the capacitor at time $t = 0$, and at some later time t the current in the circuit is $i(t)$ and the charge on the capacitor plates is $q(t)$. As we watch for a short time dt, a small charge dq passes through the battery, and the same amount of charge passes through the resistor and is stored on the capacitor plates.

A) How much energy dU_{ε} has been supplied to dq by the battery?

B) What is the rate at which the resistor is dissipating heat?

C) What is the differential energy loss dU_R in the resistor in the time dt?

D) How much energy U_C is stored in the capacitor at this moment?

E) What is the differential increase dU_C in this energy due to the addition of dq?

F) Write the equation that states that the energy supplied by the battery is equal to the energy dissipated by the resistor and the energy stored in the capacitor.

G) Divide the equation from part F) by dt and simplify. Verify that the resulting equation follows from the Loop Rule.

H) Write the equation from part G) as a differential equation in q.

I) Integrate the differential equation in part H) to find an expression for $q(t)$.

J) Given your expression for $q(t)$, develop the expression for $i(t)$.

K) This is the equation of the form $i = ae^{-bt}$ from the capacitor lab. What is the constant a, and what does it represent?

L) If the capacitor were replaced by a wire, what steady current would the ammeter measure?

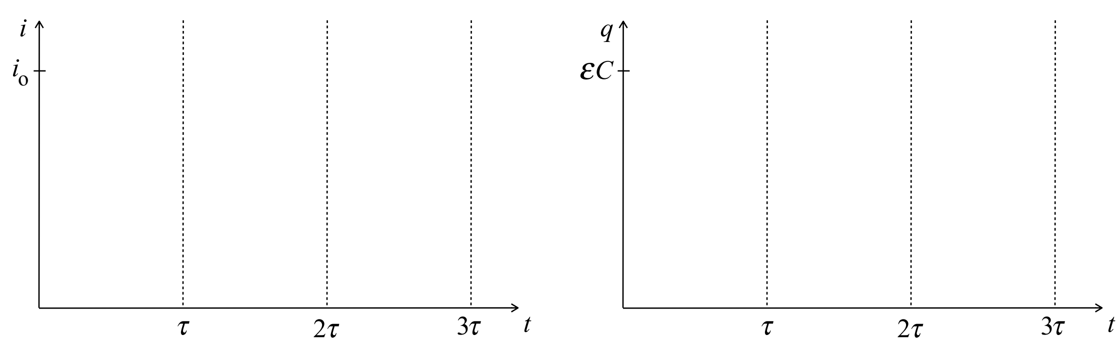
M) Make a general statement about the initial current in an *RC* circuit.

N) Show that the quantity *RC* has units of time:

O) The quantity *RC* is called the *time constant* τ (tau) of the *RC* circuit. What fraction of the initial current remains after the following times? τ: 2τ: 3τ:

P) For any time t, what is the ratio $\dfrac{i(t)}{i(t+\tau)}$?

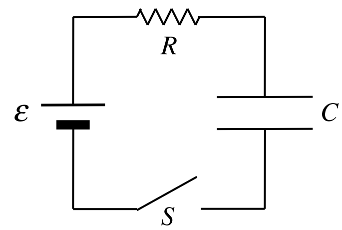
Q) On the axes below, left, sketch a graph of $i(t)$ for the resistor in an *RC* circuit. On the axes below, right, sketch a graph of $q(t)$ for the capacitor in an *RC* circuit. Make your graphs approximately to scale.



R) Use these graphs to describe how the potential differences across the resistor and the capacitor vary with time in an *RC* circuit.

Example

The series circuit shown to the right contains a resistance $R = 2\times10^{6}\ \Omega$, a capacitor of unknown capacitance C, and a battery of unknown emf $\mathcal{E}$ and negligible internal resistance. Initially the capacitor is uncharged and the switch S is open. At time $t = 0$ the switch S is closed. After this time, the current in the circuit is described by the equation



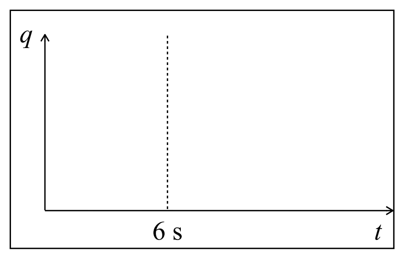
$$i(t) = (10\ \mu\text{A})e^{-t/6\text{s}}$$

A) Determine the emf of the battery.

B) By integrating with appropriate limits, develop an expression for the charge on the capacitor as a function of time.

C) On the axes to the right, sketch a graph of the charge q on the capacitor as a function of time t. Indicate significant points on the q axis.

D) Determine the capacitance C.

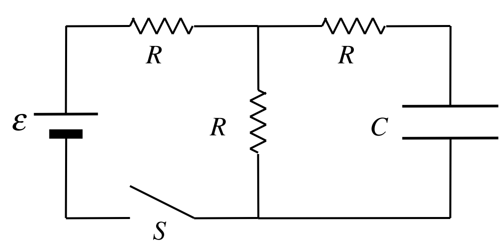


Non-simple *RC* Circuits

If the *RC* circuit is not a single capacitor in series with a resistor and a battery, the analysis is more complex, but still uses the concepts of energy transfer and the Loop Rule. Often it is sufficient to understand the initial behavior (when the capacitor behaves like a wire) and the behavior after a long time (when the capacitor behaves like an open switch).

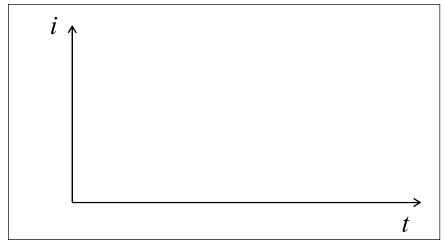
Consider the circuit to the right.

A) What is the current through the battery immediately after the switch is closed?



B) What is the current through the battery a long time after the switch is closed?

C) On the axes to the right, sketch the current in the battery as a function of time, showing initial and final values.



D) Determine the potential difference V_C across the capacitor plates after a long time.

E) Determine the charge Q that is stored on the capacitor plates after a long time.

After being closed for a long time, the switch is opened. We reset the stopwatch to $t = 0$.

F) Of the three resistors in the circuit (left, middle, right), which will have current at the instant the switch is opened? Explain your reasoning.

G) What is the direction in which current flows once the switch is opened? What is the initial value of this current?

At some later time t, the current in the circuit is $i(t)$, and the charge left on the capacitor is $q(t)$.

H) Use the Loop Rule to write an equation relating i and q to the capacitance C and the resistance R.

I) Write the above equation as a differential equation in q. Note that since charge is *leaving* the capacitor, $i = -\frac{dq}{dt}$.

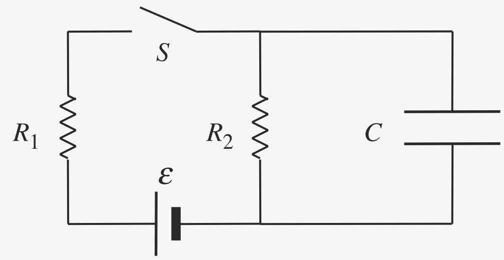
J) Separate variables and integrate with appropriate limits to determine $q(t)$.

K) Given your expression for $q(t)$, develop the expression for $i(t)$.

In general, we can assume that the current in an RC circuit starts at the initial value and asymptotically approaches the final value.

Examples

1. In the circuit shown to the right, the switch S is open and the capacitor C is initially uncharged. The values of the resistances, capacitance and emf are shown.



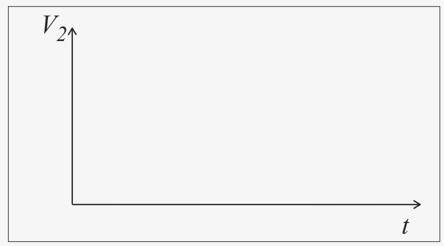
$C = 5\ \mu\text{Fd}$ $R_1 = 5\ \text{k}\Omega$

$\mathcal{E} = 3\ \text{kV}$ $R_2 = 10\ \text{k}\Omega$

A) Calculate the current in the resistor R_1 immediately after the switch S is closed.

B) Calculate the current i_2 in the resistor R_2 after the switch has been closed for a long time.

C) On the axes to the right, sketch the potential difference V_2 across R_2 as a function of time t indicating initial and final values. Show your calculation.



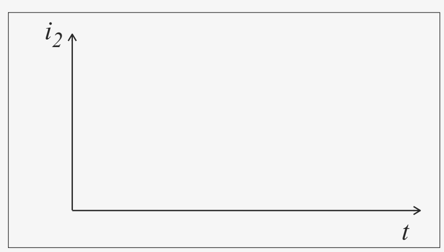
D) Calculate the charge Q on the capacitor after a long time.

E) Calculate the energy U stored in the capacitor after a long time.

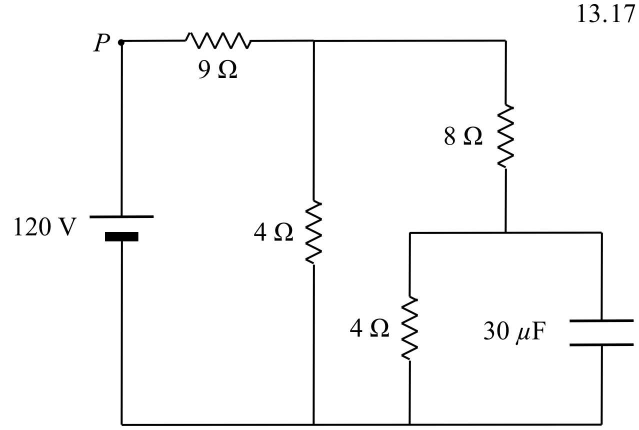
Now the switch S is open after being closed for a long time.

F) On the axes below, right, sketch the current i_2 in the resistor R_2 as a function of time t, and indicate the initial and final values.

G) Is the current through R_2 *up* or *down*? Explain.



2. In the circuit shown to the right, the battery has been connected for a long time so that the currents have steady values. Given these conditions, calculate each of the following:



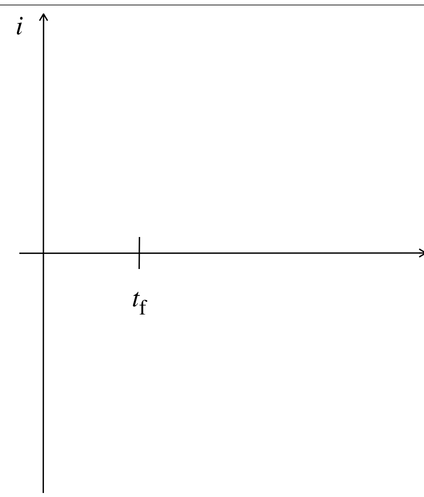
A) The current in the 9 Ω resistor

B) The potential difference across the capacitor

C) The energy stored in the capacitor

At time t_f, the connection at point P fails, and the current through the battery becomes zero.

D) On the axes to the right, sketch a graph of the current in the 8 Ω resistor as a function of time both before and after time t_f, clearly indicating current values on the i-axis. Let current downward through the resistor be positive. Show your calculations below.

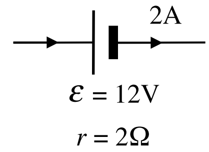


AP Physics C
Unit 13 Practice Test

Name ___________________________________

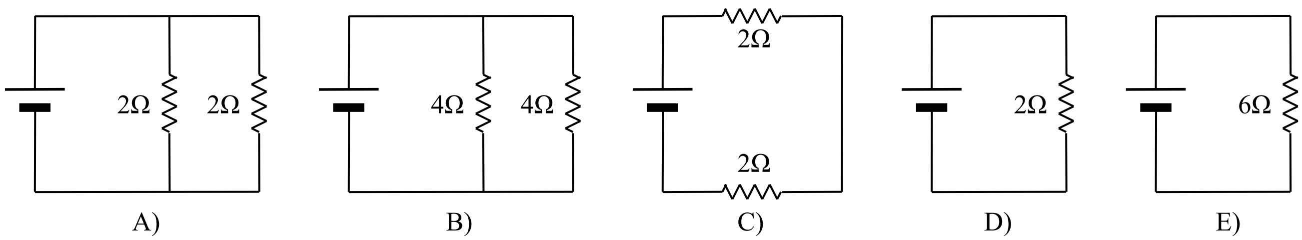
Part I
Multiple Choice

1. A 12-volt storage battery, with an internal resistance of 2 Ω, is being charged by a current of 2 amperes as shown in the diagram to the right. Under these circumstances, a voltmeter connected across the terminals of the battery will read



A) 4 volts
B) 8 volts
C) 10 volts
D) 12 volts
E) 16 volts

For questions 2 - 4: The batteries in each of the circuits shown below are identical, and they and the wires have negligible resistance.

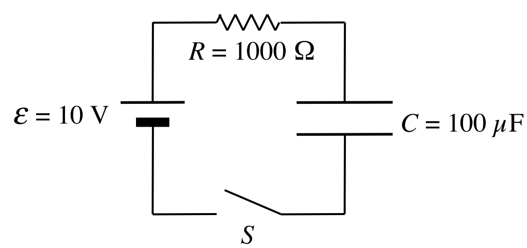


2. In which circuit is the current furnished by the battery the greatest?

3. In which circuit is the equivalent resistance connected to the battery the greatest?

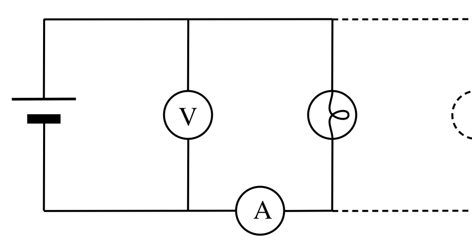
4. Which circuit dissipates the least power?

5. In the *RC* circuit to the right, a battery of emf 10 volts and no internal resistance is connected in series with a 1000 Ω resistor and a 100 μF capacitor. About how long will it take after the switch is closed for the current to become less than 1 milliampere?



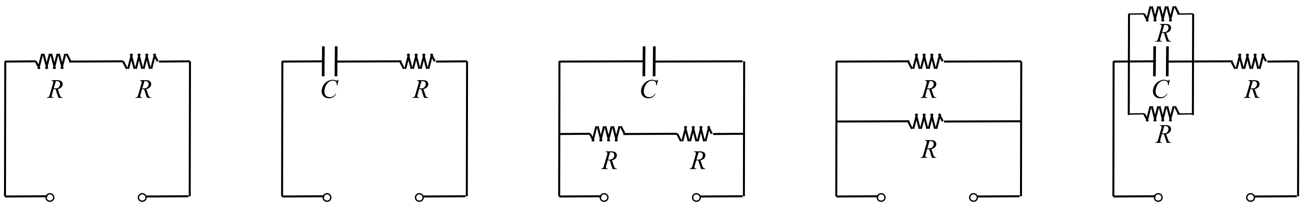
A) between t = 0 and one time constant
B) between one and two time constants
C) between two and three time constants
D) between three and four time constants
E) more than four time constants

6. A lamp, a voltmeter V, an ammeter A, and a battery with no internal resistance are connected as shown to the right. Connecting another lamp as shown by the dashed lines would



A) increase the ammeter reading
B) decrease the ammeter reading
C) increase the voltmeter reading
D) decrease the voltmeter reading
E) produce no change in either meter reading

Questions 7 and 8 relate to the five incomplete circuits below, composed of resistors R, all of equal resistance, and capacitors C, all of equal capacitance. A battery that can be used to complete any of the circuits is available.

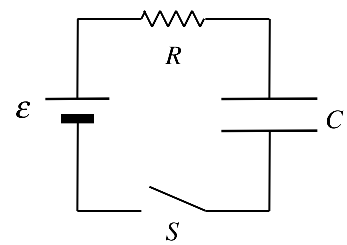


A) B) C) D) E)

7. Into which circuit should the battery be connected to obtain the greatest steady power dissipation?

8. Which circuit will retain stored energy if the battery is connected to it for a while and then disconnected?

9. In the circuit shown to the right, the capacitor is initially uncharged. At time $t = 0$, switch S is closed. Which of the following is true at time $t = RC$?



   A) The current is $\dfrac{\varepsilon}{eR}$.

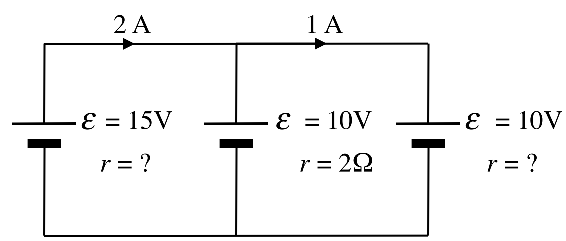
   B) The current is $\dfrac{\varepsilon}{R}$.

   C) The voltage across the capacitor is ε.

   D) The voltage across the capacitor is $\dfrac{\varepsilon}{e}$.

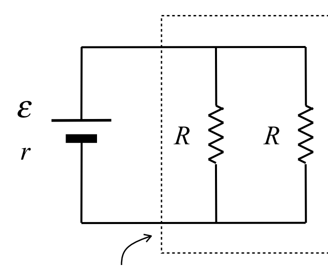
   E) The voltages across the capacitor and resistor are equal.

10. In the circuit shown to the right, the emf's of the batteries are given, as well as the currents in the outside branches and the internal resistance of the middle battery. The internal resistance of the 15V battery must be



   A) 1 Ω
   B) 1.5 Ω
   C) 2 Ω
   D) 3 Ω
   E) 4.5 Ω

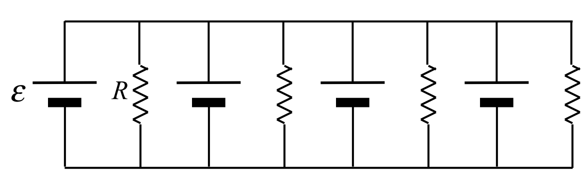
11. A battery having emf ε and internal resistance r is connected to a circuit consisting of two parallel resistors each having resistance R. For what value of R will the power dissipated by the external circuit be a maximum?



External circuit

   A) 0
   B) $r/2$
   C) r
   D) $2r$
   E) $4r$

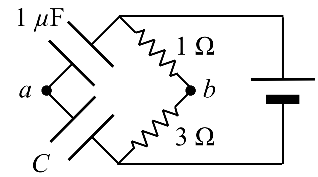
12. In the circuit to the right, all batteries are identical with emf ε and no internal resistance, and all resistors are identical with resistance R. The total power dissipated by the resistors is



   A) $\dfrac{\varepsilon^2}{4R}$ B) $\dfrac{\varepsilon^2}{R}$ C) $\dfrac{2\varepsilon^2}{R}$ D) $\dfrac{16\varepsilon^2}{R}$

   E) None of the above

13. In the circuit shown to the right, the potential difference between points *a* and *b* is zero when the capacitance *C* is

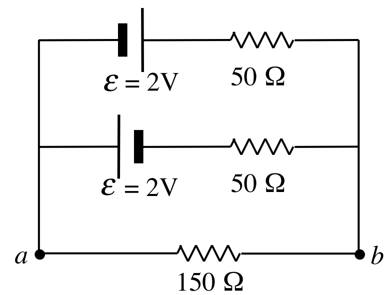


A) 1/3 μF
B) 2/3 μF
C) 2 μF
D) 3 μF
E) 9 μF

14. When two identical resistors are connected in series to an ideal battery, the total power dissipated is *P*. When the same two resistors are connected in parallel to the same battery, the total power dissipated is

A) 1/4 *P*
B) 1/2 *P*
C) *P*
D) 2 *P*
E) 4 *P*

15. In the circuit shown to the right, the current in each battery is 0.04 A. The potential difference between points *a* and *b* is



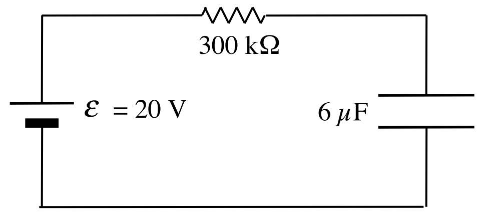
A) 0 volts
B) 2 volts
C) 4 volts
D) 6 volts
E) 8 volts

1. E 2. A 3. E 4. E 5. C 6. A 7. D 8. B 9. A 10. B 11. D 12. E 13. A 14. E 15. A

Part II

Show your work

Credit depends on the quality and clarity of your explanations



1. A battery with $\mathcal{E} = 20$ V is connected in series with a resistor of 300 kΩ and a parallel-plate capacitor with $C = 6\ \mu$F.

   A) Determine the energy stored in the capacitor when it is fully charged.

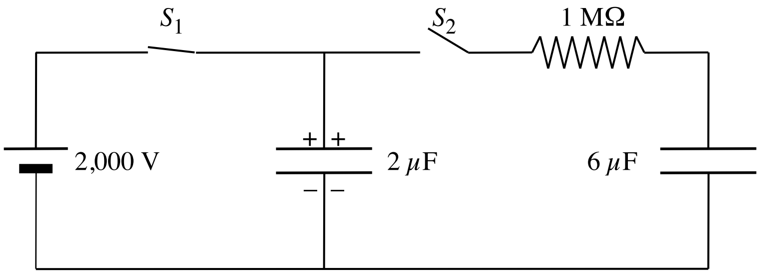
   The spacing between the capacitor plates is suddenly increased (in a time short compared to the time constant of the circuit) to four times its original value.

   B) Determine the work that must be done in increasing the spacing in this fashion.
   C) Determine the current in the resistor immediately after the spacing is increased.

   After a long time, the circuit reaches a new static state.

   D) Determine the total charge that has passed through the battery.
   E) Determine the energy that has been added to the battery.

2. The 2 μF capacitor shown in the circuit to the right is fully charged by closing switch S_1 and keeping switch S_2 open, thus connecting the capacitor to the 2,000 volt power supply.



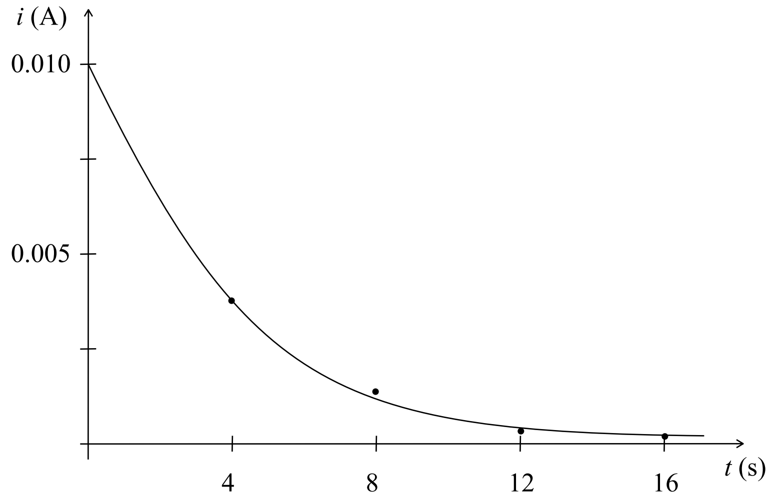
A) Determine each of the following for this fully charged capacitor.
   i) The magnitude of the charge on each plate of the capacitor
   ii) The electrical energy stored in the capacitor

At a later time, switch S_1 is opened. Switch S_2 is then closed, connecting the charged 2 μF capacitor to a 1 MΩ (1×10^6 Ω) resistor and a 6 μF capacitor, which is initially uncharged.

B) Determine the initial current in the resistor the instant after switch S_2 is closed.

Equilibrium is reached after a long period of time.

C) Determine the charge on the positive plate of each of the capacitors at equilibrium.
D) Determine the total energy stored in the two capacitors at equilibrium. If the energy is greater than the energy determined in part A) ii), where did the increase come from? If the energy is less, where did the electrical energy go?

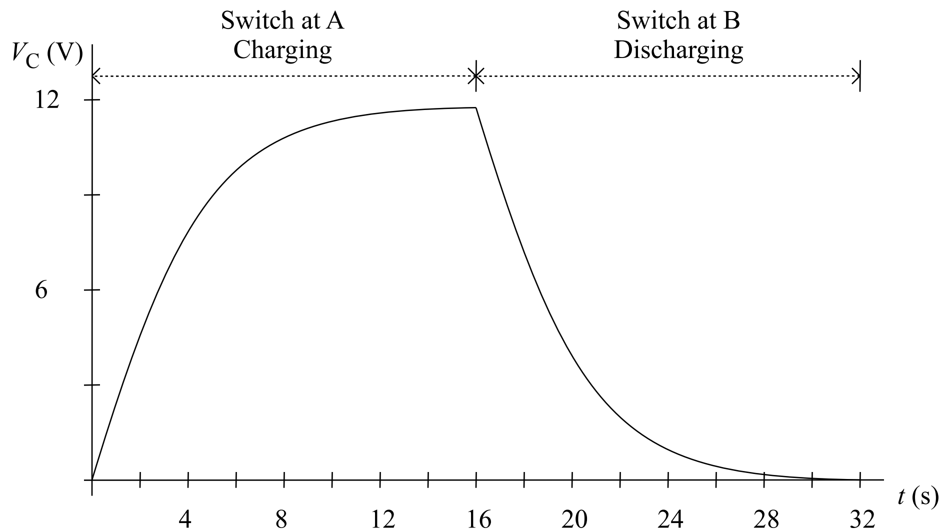


3. In the laboratory, you connect a resistor and a capacitor with unknown values in series with a battery of emf $\mathcal{E}$ = 12 V. You include a switch in the circuit. When the switch is closed at time $t = 0$, the circuit is completed, and you measure the current through the resistor as a function of time as plotted above. You use Logger Pro to find that the current decays according to the relation $i(t) = \frac{\mathcal{E}}{R} e^{-\frac{t}{4\text{ s}}}$.

   A) Using common symbols for the battery, the resistor, the capacitor, and the switch, draw the circuit that you constructed. Show the circuit before the switch is closed and include whatever other devices you need to measure the current through the resistor to obtain the above plot. Label each component in your diagram.
   B) Having obtained the curve shown above, determine the value of the resistor you placed in this circuit.
   C) What capacitance did you insert in the circuit to give the result above?

(continued on next page)

You are now asked to reconnect the circuit with a new switch in such a way as to charge and discharge the capacitor. When the switch in the circuit is in position A, the capacitor is charging, and when the switch is in position B, the capacitor is discharging, as represented by the graph below of voltage V across the capacitor as a function of time.



D) Draw a schematic diagram of the RC circuit that you constructed that would produce the graph above. Clearly indicate switch positions A and B on your circuit diagram and include whatever other devices you need to measure the voltage across the capacitor to obtain the above plot. Label each component in your diagram.

AP Physics C
Unit 14 Name____

Special Relativity

Albert Einstein's Special Theory of Relativity has to do with the perception of events from different reference frames. An *inertial* reference frame is one which is not rotating or accelerating. Einstein's assumptions are:

1. The laws of physics are the same in any inertial reference frame.
2. The speed of light in vacuum (c) is the same to all observers in inertial reference frames, regardless of their motion relative to each other or the source of light.

Given these assumptions, Einstein showed that *time* and *length* are not absolute quantities, measured the same by all observers, but *relative* quantities, whose observed values depend on the motion of the observer relative to the object.

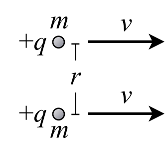
If v is the speed of the observer relative to the object, then the change in time and length depend on the factor $\dfrac{1}{\sqrt{1-v^2/c^2}}$.

This factor has a special name (the *Lorentz Factor*), and a special symbol (γ , the Greek letter *gamma*). The Lorentz Factor is a pure number, whose value ranges from 1 (when $v = 0$), to infinitely big (as v approaches c).

The time intervals in one reference frame *expand* by the Lorentz Factor when observed from another frame. This is usually explained in terms of the "ticks" of a clock. Let Δt_o represent the time interval between ticks of a clock in a reference frame at rest. If that same clock moves with a speed v, then the observed time interval Δt would be *longer* by the Lorentz Factor: $\Delta t = \gamma \Delta t_o$. *Dilation* means expansion, so this phenomenon is known as *time dilation*.

The distance between two points measured in one reference frame become *shorter* (contracted) when viewed from a reference frame in relative motion. If L_o represents the distance measured in a reference frame at rest, then the distance L measured in a moving frame is shorter by the Lorentz Factor; that is, it is proportional to $1/\gamma$: $L = \dfrac{L_o}{\gamma}$. This is known as *length contraction*. Length contraction happens only in the direction of motion, not in perpendicular directions.

Forces on Moving Charges

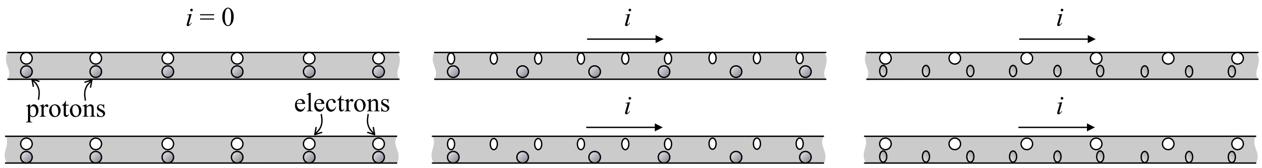


Consider two equal positive charges $+q$, each with mass m, a distance r apart, moving together at the same speed v in the laboratory reference frame (neglect gravitational forces). What is the force of electrical repulsion between these charges? $F_E =$ This repulsive force would accelerate the charges away from each other. In a reference frame moving with the charges, they appear to be at rest; in this reference frame, what is the acceleration of each charge at the moment they are a distance r apart? $a =$ In the laboratory frame, would time dilation make this acceleration *greater than a*, or *less than a*? Explain.

In the laboratory frame, would the force acting between the charges be *greater* or *less* than F_E above?

In the laboratory frame, there must be another force acting in addition to F_E to account for the observed acceleration. Is the direction of this force on each charge *toward* or *away from* the other charge? Note that this force only acts on *moving* charges, and the faster they move, the greater the force. How fast would the particles have to move so that this force completely cancels F_E? Explain using the Lorentz Factor.

Forces on Current-Carrying Wires

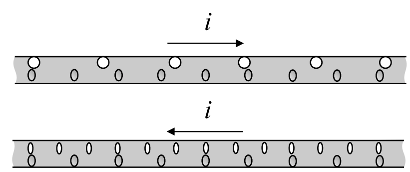


Laboratory frame　　Proton frame　　Electron frame

The diagrams above represent an idealized view of two parallel wires. When there is no current flowing in the wires, there are equal numbers of protons and electrons, so that the positive and negative linear charge densities $\pm\lambda_o$ are equal. Now consider sending equal currents to the right in both wires. In the lab frame (the proton frame), the electrons move. Which way do the electrons move? In the proton frame, the electrons are *length contracted* as shown in the second diagram. In this frame, is the negative charge density of each wire *greater than* or *less than* λ_o?

How would the protons in one wire respond to the other wire?

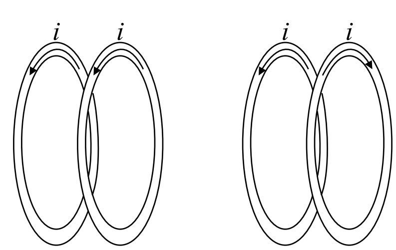
In the electron frame, the *protons* are length contracted as shown in the third diagram. In this frame, is the positive charge density of each wire *greater than* or *less than* λ_o? How would the electrons in one wire react to the other wire? If the wires carry current in opposite directions, then in the frame of the electrons in one wire, the electrons in the other wire will be length contracted *more* than the protons are. How would the electrons in the top wire react to the protons and electrons in the bottom wire? Explain.



Frame of electrons in top wire

Make a general statement about the force between wires that carry current in the *same* direction and wires that carry current in the *opposite* direction.

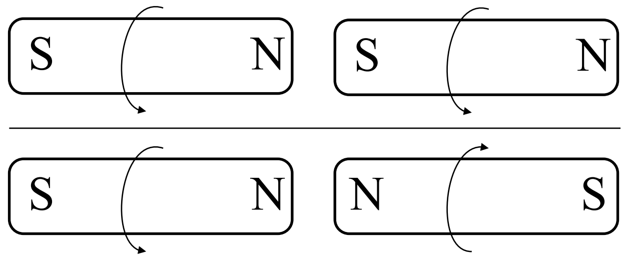
Two wires are bent into circular loops, and carry current in the same direction as shown in the first diagram to the right. What would happen to these two loops?



If the currents in the loops were in opposite directions, as in the second diagram, what would happen to the loops?

Forces Due to Electron Orbits

Electrons in their orbits can be considered tiny current loops like the ones on the previous page. In normal materials there are so many electrons and their orbits are so randomly oriented that there is no net effect. However, in some materials such as iron, it is possible to orient more of the orbits one way than another.

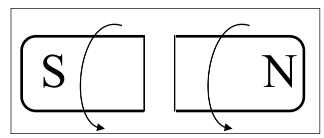


Imagine that we have two bars of iron in which we have oriented the electron orbits predominantly the same way, as shown in the top diagram. We have randomly chosen two letters, “S” and “N,” to help keep track of the orientations in the two bars. What is the direction of the force on each bar due to the other?

If the bar on the right is flipped around as shown, what is the direction of the force on each bar due to the other?

In ancient Greece, stones were discovered that behaved like the bars above. They were eventually called *magnets* after the Greek region called Magnesia. The ends of the bars behave in a way that reminds us of electric charges; “likes repel and unlikes attract.” In this sense, bar magnets like the ones above are like *dipoles*.

If one were to cut a bar magnet in half, as shown to the right, would you expect those halves to *attract* or *repel*? Explain, and indicate the appropriate letter at each end of the new bars.

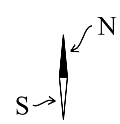


A single pole, either an “N” or an “S,” would be called a *magnetic monopole*. Do you think it would be possible to isolate a magnetic monopole? Explain how or why not.

Magnetic Field

In our discussion of electricity and electric field, we considered an electric charge as causing an electric field in its vicinity, such that other electric charges placed in that field feel an electric force. We drew vectors and field lines to represent the magnitude and direction of $\vec{E}$. In a similar way, we’d like to examine the region around a bar magnet to see what the effect would be on other bar magnets in the region. We use the symbol $\vec{B}$ for magnetic field.

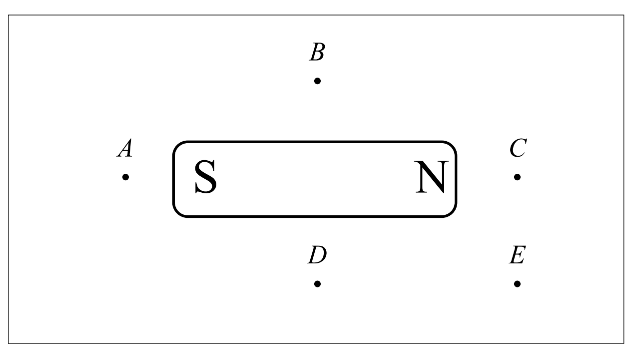
To investigate the magnetic field of a bar magnet, we use another, small bar magnet that can rotate freely. Such an instrument is called a *compass* as shown to the right. In the space to the below, draw compasses at the lettered points to show the orientation of the compass if it were placed there.



Compass

Download the PhET simulation Magnet and Compass from the web site. Move the simulated compass to the corresponding points to check your predictions.

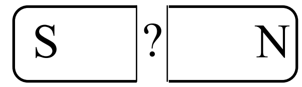
In the simulation, small compasses are drawn at regular intervals to show the direction of the magnetic field in the whole region. The strength of the field is represented by how bright or faded the compasses are.



On the diagram to the right, sketch field lines that show the general direction of the magnetic field in the region. The field lines should be closer together where the field is stronger, and farther apart where it is weaker. We arbitrarily choose the direction in which the N pole points to be the direction of the magnetic field. Draw arrowheads on your field lines to indicate the direction of the field.

Compasses have been used for centuries to guide travelers on land and sea. On the simulation, check the "Show planet Earth" checkbox. Move the compass along the surface of the Earth and explain why the letters "S" and "N" were chosen to label the poles of magnetic dipoles, and why the Earth's magnetic poles are labeled as they are.

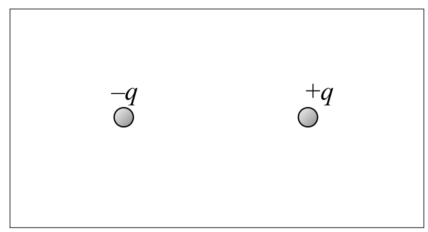
Consider the magnet broken in half from the previous page. If a small compass were placed between the pieces, which direction would it point? Would you expect the field there to be strong or weak? Explain.



These pieces can be held arbitrarily close together. What does this say about the magnetic field *inside* the original magnet? Is it strong or weak? What direction does it point?

On the simulator, check the "See Inside Magnet" checkbox. Verify that your prediction about the strength and direction of the field inside the magnet is correct. Complete the diagram of field lines on the previous page by drawing field lines inside the magnet.

Magnetic dipoles are similar in many ways to electric dipoles, but there is a major difference. On the diagram to the right, sketch a few electric field lines in the region of the electric dipole. Look in particular at the region *between* the poles. What is the major difference between the fields of the two dipoles?



Gauss' Law for Magnetism

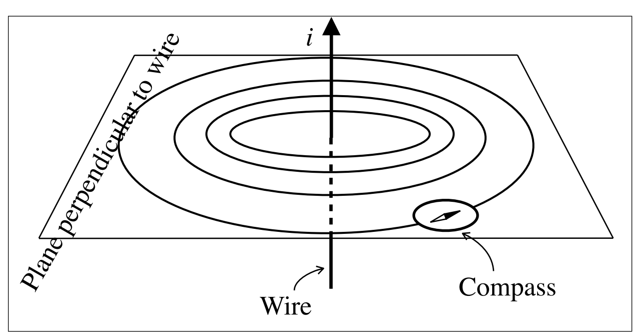
Electric field lines start on positive charges and end on negative charges. Because there are no magnetic monopoles, magnetic field lines don't start or end; instead, they form closed loops. Consider a closed Gaussian surface in a region of space where there are magnetic field lines. What would you expect to be true about the flux of the magnetic field over this surface? Explain.

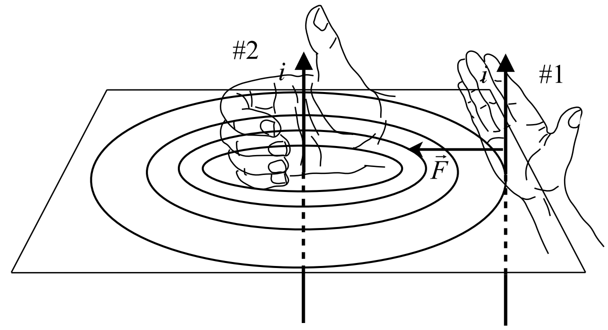
In view of the above, write Gauss' Law for magnetism:

Magnetic Field and Magnetic Force

We have seen that there is a magnetic force between two parallel current-carrying wires. This means that the first wire causes a magnetic field, and the second wire experiences a force due to this field, and vice versa. We can examine the magnetic field of a wire using a compass. If we did so, we would find that in a plane perpendicular to the wire, the magnetic field lines would form circular loops around the wire.

On the diagram, draw arrowheads on the field lines to show the direction of the field indicated by the compass.





Right-Hand Rules

If we place a second wire in the magnetic field of the first wire, carrying the same current in the same direction, we know that it will experience a force toward the first wire. This means that the magnetic *force* is perpendicular to the magnetic *field* as defined by the compass direction, and it is also perpendicular to the *current* that experiences the force.

To remember the relative directions of the current, the magnetic field and the magnetic force, we have two right-hand rules:

#1: Force on a current or moving positive charge: With your thumb in the direction of the current (or velocity of positive charge), and your fingers in the direction of $\vec{B}$, your palm will "push" in the direction of the force.

#2: Field of a current in a wire: With your thumb in the direction of the current, wrap your fingers around the wire. This shows the direction of $\vec{B}$ in circular loops around the wire.

Note that Right-Hand Rule #1 applies to (positive) current or moving positive charge. Moving negative charge experiences a force in the opposite direction; we can say that moving negative charge obeys Left-Hand Rule #1.

Magnitude of $\vec{B}$

We have used the term "charge" in reference to the electric force, but it can be used more generally to refer to a property of an object that makes it respond to a particular field; ie., *electric charge* is what responds to *electric field*. In this sense, what is *gravitational charge*? Note that the "charge" that responds to a field is the same as the "charge" that causes the field.

We have seen that the magnetic force occurs between charges only when they are in motion relative to an observer. *Magnetic charge*, the property that causes and responds to a magnetic field, is the product of the charge q on the particle and the speed v at which it moves. However, the direction of the particle's velocity must be perpendicular to the direction of $\vec{B}$ for it to feel the maximum magnetic force. If it's not, then the force is proportional to the component that *is* perpendicular to $\vec{B}$. We call this component $v_\perp$.

We determine the magnitude of $\vec{B}$ at a point in the same way we find the magnitude of other fields: Consider a "charge" at that point, and divide the force felt by the "charge" by the magnitude of the "charge." So the magnitude of $\vec{B}$ can be written:

$B = \dfrac{F}{qv_\perp}$. What are the SI units of B? One of these is called a *Tesla* (T) after Nikola Tesla, an electrical engineer and inventor. We can solve for F: $F = Bqv_\perp$, so a charge of 1 C, moving at 1 m/s perpendicular to a 1 T magnetic field will experience a 1 N force.

The equation above for the magnetic force involves three vector quantities ($\vec{F}$, $\vec{B}$, and $\vec{v}$) and one scalar (q). Write the equation for $\vec{F}$ using vector cross product notation so that Right-Hand Rule #1 is satisfied:

Of the three vectors in this equation, which pair(s) *must* be at right angles, and which pair(s) can be at arbitrary angles?

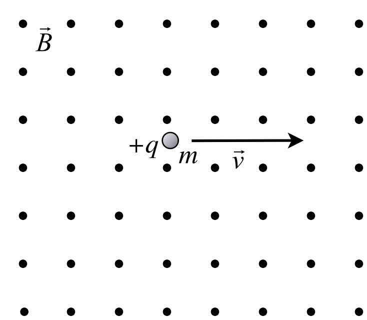
Since we often have to draw vectors in three dimensions on two-dimensional paper, we have a convention for drawing vectors perpendicular to the page. We think of the vector as an arrow. "•" (the tip of the arrow) represents a vector coming out of the page, and " ×" (the tail feathers of the arrow) represents a vector going into the page.

Examples

1. You are sitting in a room with a uniform magnetic field. You have your back to the back wall, and an electron beam, moving horizontally from the back wall to the front wall, is deflected to your right. What is the direction of $\vec{B}$?

2. A uniform magnetic field $\vec{B}$ with magnitude 1.2×10^{-3} T points vertically upward in a room. A proton with a mass of 1.67×10^{-27} kg and an energy of 5.3 MeV moves horizontally from south to north past a certain point in the room. An eV is an "elementary charge•volt" and the elementary charge is 1.6×10^{-19} C. 1 MeV = 10^6 eV
   A) What is the energy of the proton in Joules?
   B) What is the speed of the proton?
   C) What is the magnitude of the magnetic force on the proton?
   D) What is the direction of the force on the proton?
   E) What is the acceleration of the proton?

3. The diagram to the right shows a particle with mass m and positive charge $+q$ moving to the right with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$ directed out of the page.



   A) What is the magnitude and direction of the magnetic force $\vec{F}_B$ on the charge?

   This force will deflect the path of the particle, but since the magnetic force remains perpendicular to $\vec{v}$, the particle moves along a circular path at constant speed.
   B) Determine the radius r of this circular path.
   C) How does the radius of the path depend on
      i) The magnitude of $\vec{B}$?
      ii) The particle's momentum?
      iii) The particle's kinetic energy?

D) Determine the period T of the particle's circular motion.

E) How does the period depend on

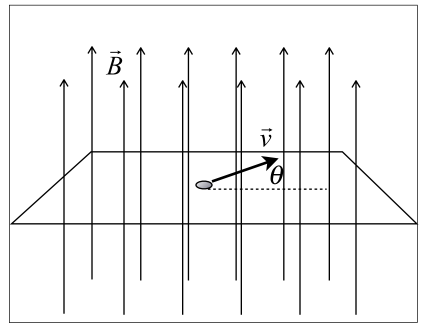
i) The magnitude of $\vec{B}$?

ii) The particle's mass?

iii) The particle's charge?

iv) The radius of the path?

Instead, the particle moves so that its velocity is not perpendicular to $\vec{B}$, but at an angle θ to the plane of the page as shown to the right in the side view.



F) Sketch the path of the particle on the diagram.

This path is called a *helix*, a three-dimensional curve that wraps around a cylinder. It can be described in terms of its *axis*, *radius*, and *pitch*. The axis and radius correspond to those of the cylinder. The pitch is the distance between successive wraps of the helix, measured parallel to the axis.

G) Describe the direction of the axis of the helical path with respect to the direction of $\vec{B}$.

H) Determine the radius of the helical path.

I) Determine the period (the time to complete one wrap).

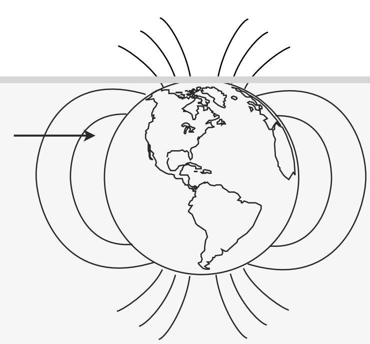
J) Determine the pitch of the helical path.

K) If the particle continues to move along a helical path, but the field gets *stronger*, how would that affect

i) the radius of the helix?

ii) the centripetal acceleration of the particle?

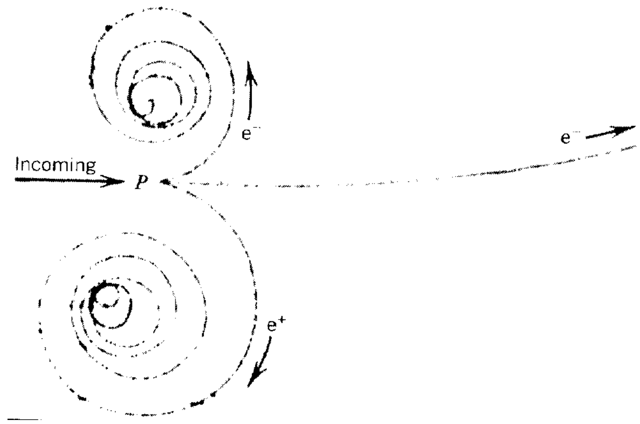
When charged particles accelerate, they radiate electromagnetic waves. The greater the acceleration, the higher the energy they radiate. The sun emits charged particles that travel to the earth and encounter its magnetic field as shown to the right.



L) Describe what happens to such a particle in the northern hemisphere.

Go to the web site and play the video clip "ISS Aurora.mp4" to see a view of this from space.

4. The figure to the right shows the tracks of two electrons (e^-) and a positron (e^+) in a bubble chamber emerging from a collision at point *P*. A positron is identical to an electron, but with a positive charge. There is a magnetic field filling the chamber, perpendicular to the plane of the page. The tracks are made as the particles interact with the supersaturated gas to make bubbles.

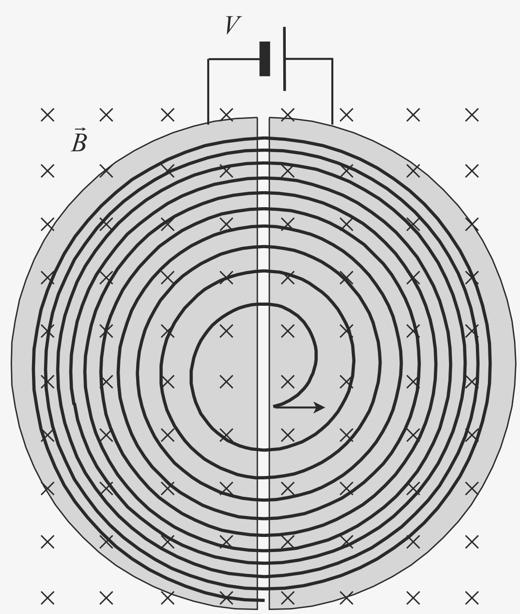


A) Why are the tracks spirals and not circles?

B) Why do the two electrons make different tracks?

C) What is the direction of the magnetic field?

5. (<u>The Cyclotron</u>) The diagram to the right shows two D-shaped electrodes called "dees" with a potential difference *V* between them. The system is immersed in a uniform magnetic field $\vec{B}$ into the page. A particle with mass *m* and charge +*q* is inserted with a small initial speed v_0 near the center as shown. This device is called a *cyclotron.*



A) Determine the time it takes for the particle to complete the semicircle in the right dee.

The particle is accelerated by the potential difference in the gap and enters the left dee.

B) What is the energy it gains in the gap?

C) Is the time it takes to complete the semicircle in the left dee *greater than*, *less than*, or *the same as* the time it took in the right dee? Explain.

While the particle is in the left dee, the polarity of the potential difference is reversed. When the particle reaches the gap, it is accelerated again. As long as the polarity of the potential is reversed with just the right frequency, this process continues until the particle reaches the outer edge, gaining energy every time it crosses the gap.

D) If the radius of the dees is *R*, determine the maximum speed of the particle.

Assume the particle is a proton, with a mass $m_p = 1.67\times10^{-27}$ kg and a charge 1.6×10^{-19}C. If the magnetic field is 0.1 T, the accelerating potential is 1,000 V, and the radius *R* is 5 m:

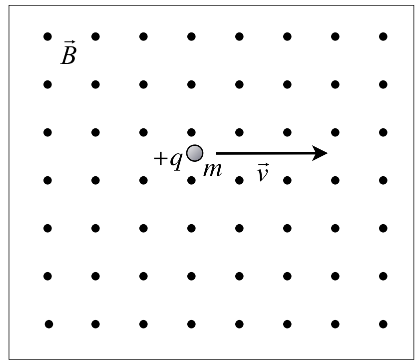
E) Determine the maximum speed of the proton

F) Determine the energy of the proton in MeV.

G) Determine the number of times the proton crossed the gap.

H) Determine the frequency with which the polarity of the potential was reversed.

6. (The Velocity Selector) Consider again the particle with mass m and positive charge $+q$ moving perpendicular to the magnetic field $\vec{B}$.



A) What is the direction of the magnetic force?

A uniform electric field $\vec{E}$ is created in the same region by parallel charged plates.

B) What direction should the electric field point so that there is no net force on the moving charge? Sketch the field lines on the diagram.

C) What is the magnitude of $\vec{E}$ if there is no net force on the moving charge?

A beam of charged particles with different masses, charges (both positive and negative), and speeds enters the region above with $\vec{B}$ and $\vec{E}$ fields. Particles with mass m, charge $+q$, and speed v experience no net force, so they move along a straight line.

D) Describe what happens to the following particles:

i) Particles with mass m, speed v and charge $-q$:

ii) Particles with mass m, speed v and positive charge greater than $+q$:

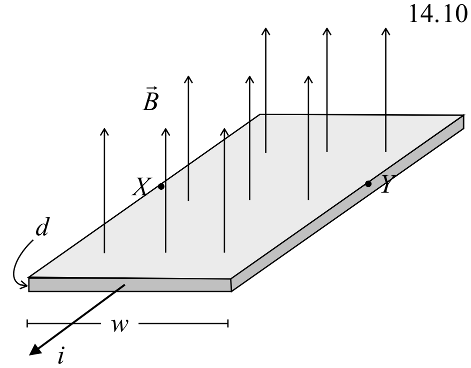
iii) Particles with speed v and charge $+q$ but mass greater than m:

iv) Particles with mass m and charge $+q$ but speed greater than v:

E) Explain why this device is called a *velocity selector*.

F) The magnetic field is removed, but the electric field remains. Describe the path of the particle. Explain.

7. (The Hall Effect) In 1879, electrons hadn't been discovered, and it was unclear what particles were responsible for carrying the moving charge that constituted current. Edwin Hall devised an experiment to determine the sign of the charge carriers. Consider a wide, flat conductor carrying conventional current in the direction shown. Create a magnetic field perpendicular to the conductor.



A) If the charge carriers are positive, are they traveling *toward* or *away from* you in the diagram?

B) Will the magnetic force push the positive charges *toward X* or *toward Y* in the diagram?

C) The potential difference between points X and Y is measured with a voltmeter. Which side, X or Y, will be at the higher potential?

D) If the charge carriers are negative, are they traveling *toward* or *away from* you in the diagram?

E) Will the magnetic force push the negative charges *toward X* or *toward Y* in the diagram?

F) The potential difference between points X and Y is measured with a voltmeter. Which side, X or Y, will be at the higher potential?

It was discovered that in fact, charge carriers in metal conductors are negative. Recall that the drift velocity of charge carriers in a conductor is $v_\text{d} = \dfrac{J}{ne}$, where J is the current density, n is the number of charge-carriers per unit volume, and e is the elementary charge.

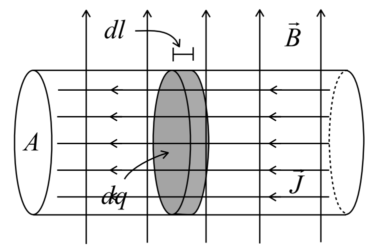
G) What is the magnitude of the magnetic force $\vec{F}_B$ on the charge carriers in the conductor?

Charge carriers crowd to one side of the conductor, leaving the opposite side with the opposite charge, creating a transverse electric field in the conductor. The force due to this electric field balances the magnetic force on the charge carriers.

H) Determine the strength of the transverse electric field, called the "Hall electric field."

I) Determine the potential difference between opposite sides X and Y of the conductor, called the "Hall potential."

Magnetic Force on a Current



We have seen that the moving charges in current-carrying wire in a magnetic field experience a force. Consider a wire of area *A* carrying a uniform current density $\vec{J}$. The wire is in a uniform magnetic field $\vec{B}$ perpendicular to the wire as shown. A small segment of current of length *dl* and charge *dq* is highlighted. This segment can be thought of as moving with the drift velocity v_d.

A) What is the magnitude and direction of the force differential force *dF* on the current segment?

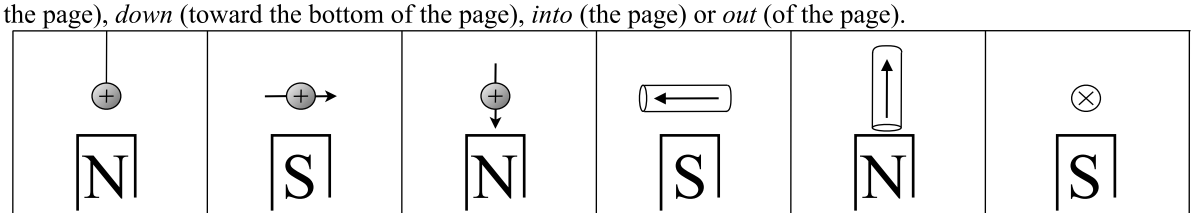
B) If *n* is the number of charge carriers per unit volume, and *e* is the elementary charge, what is the charge *dq*?

C) Express *dF* in terms of the current in the wire.

D) It is customary to define vector $d\vec{l}$ whose magnitude is the same as *dl* and whose direction is that of $\vec{J}$. If $d\vec{l}$ is *not* perpendicular to $\vec{B}$, then (as with the moving charges) the magnetic force depends on the component that is. So just as in the case of moving charges, write the above relationship as a vector cross product involving $d\vec{l}$ and $\vec{B}$.

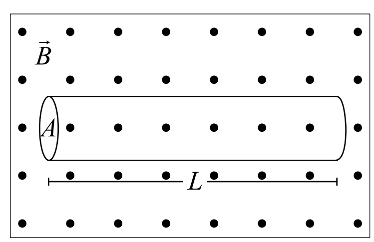
Examples

1. A bar magnet is held vertically and charges or current segments are held at or moved past a pole as shown below. For each situation, state the direction (if any) of the force on the charge or current segment as *none*, *left*, *right*, *up* (toward the top of the page), *down* (toward the bottom of the page), *into* (the page) or *out* (of the page).



	A positive charge is hung by a thread	A positive charge moves to the right	A positive charge moves down	A segment carries current left	A segment carries current up	A segment carries current into the page
Direction:						

2. There is a uniform magnetic field $\vec{B}$ pointing out of the page. You have a battery of emf ε and a copper bar, mass density δ and resistivity ρ. You are told to form a wire out of the copper such that, when connected to the battery, the magnetic force on it will equal its weight.

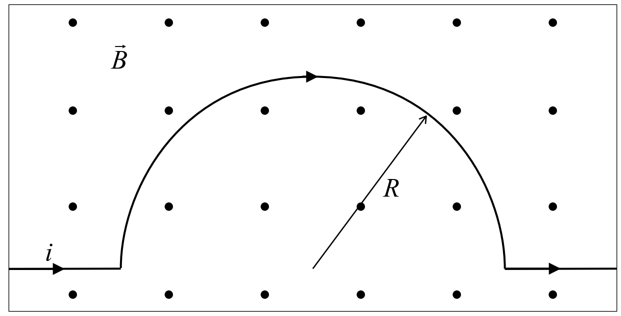


A) On the diagram, show how the battery would be connected to send current through the wire in the proper direction.

B) Find the dimensions of the wire in terms of the given quantities and constants, assuming a constant cross sectional area.

C) Check your expression in part B) for dimensional consistency.

3. Consider a wire bent into a semicircle of radius R, carrying a current i clockwise as shown to the right in a magnetic field $\vec{B}$ out of the page.



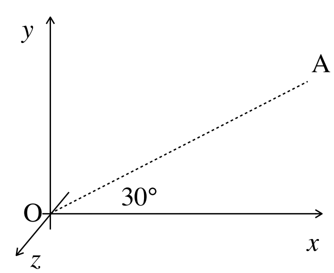
A) On the diagram, choose and label a short segment $d\vec{l}$ of the the semicircular part of the wire. Choose and label parameters that specify its exact size and location.

B) What is the direction of the differential force dF on this segment due to the magnetic field?

C) In what direction will the net force on the entire semicircular part of the wire point? Explain.

D) Integrate with appropriate limits to find the magnitude of the net force on the semicircular part of the wire.

4. Throughout a region of space, there is a uniform magnetic field $\vec{B}$. When a proton (q = 1.6×10^{-19} C) moves with a speed $v = 5\times10^{5}$ m/s away from the origin along the line OA at 30° with respect to the x-axis (in the x-y plane), it is not deflected by the $\vec{B}$ field. When a proton of the same speed moves on the x-axis in the $+x$ direction, the magnetic field exerts a force of 2×10^{-16} N on it, in the $+z$ direction (out of the page).



A) i) What is the direction of the uniform field $\vec{B}$? Explain your reasoning.

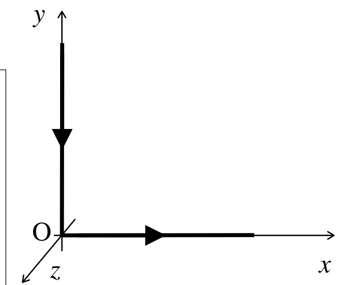
ii) Determine the magnitude of the uniform field $\vec{B}$.

B) A uniform electric field $\vec{E}$ is to be added throughout space so that the proton moving on the x-axis is not deflected.

i) What is the direction of this electric field? Explain your reasoning.

ii) Determine the magnitude of the uniform electric field $\vec{E}$.

C) A wire of length 1 m is bent at the midpoint at 90°, into the shape of an "L" and placed half along the y-axis and half along the x-axis. What is the magnitude and direction of the magnetic force on this wire if it carries 6 A down ($-y$) and right ($+x$)?



Magnetic Flux

The flux of the magnetic field is defined as for any vector field. For a given surface, the flux is $\int \vec{B} \bullet d\vec{A}$ over that surface.

What are the SI units of magnetic flux? This unit is defined as the Weber (Wb) after Wilhelm Weber, the inventor of the outdoor charcoal grill.

Examples

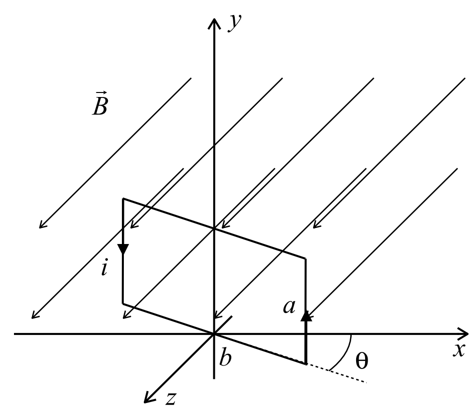
1. A rectangular loop of wire of length b and width a carries current i as shown to the right. The loop is at an angle θ to the x axis, and there is a uniform magnetic field $\vec{B}$ parallel to the z axis.

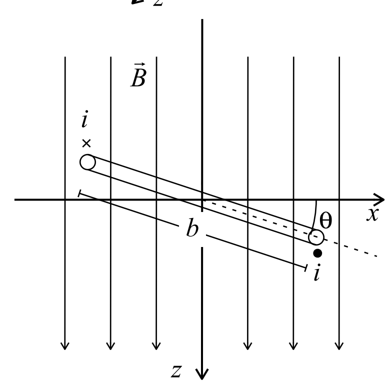
   A) Calculate the magnetic flux through the loop.

   B) Find the magnitude and direction of the torque about the y axis caused by the magnetic force on the loop.

   C) Express the torque in terms of the area of the loop.

   D) Imagine that the loop has arbitrary shape, but is in the plane of the original rectangular loop. Would the relationship in part C) still hold? Why or why not?



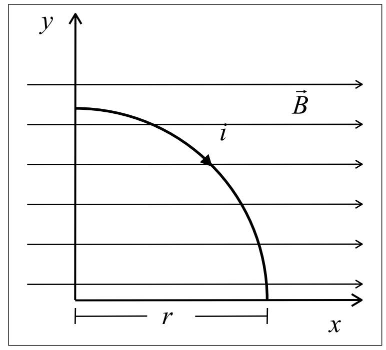


View from above

E) Does the torque on the loop tend to *increase*, *decrease*, or *not change* the magnetic flux through the loop? Explain.

F) Once the loop is in the *x*-*y* plane, do the forces on the parts of the loop tend to *increase*, *decrease*, or *not change* the magnetic flux through the loop? Explain.

2. A quarter-circular arc of wire of radius r is carrying current i as shown to the right, in a uniform magnetic field $\vec{B}$ in the positive x direction.



A) On the diagram, choose and label a short segment dl, and choose and label parameters that will specify its exact size and position.

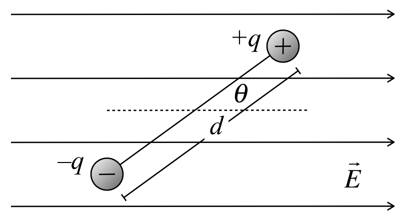
B) What is the magnitude and direction of the differential force dF on this segment?

C) What is the magnitude of the differential torque $d\tau$ about the y axis due to this segment?

D) Integrate with appropriate limits to determine the magnitude of the torque τ on the arc about the y axis. The trigonometric identity $\cos^2\theta = \frac{1}{2} + \frac{\cos 2\theta}{2}$ may prove useful.

E) Express the torque in terms of the area of the arc.

Magnetic Dipole Moment



Recall that an electric dipole experiences a torque in an electric field. The electric dipole moment $\vec{p}$ is a vector pointing from the negative to the positive charge, with magnitude qd, and we showed that the torque is given by $\vec{\tau} = \vec{p} \times \vec{E}$.

A) Using the result of Example 1 on the previous page, define the magnitude of the magnetic dipole moment $\vec{\mu}$ of a current loop so that it obeys the corresponding equation $\vec{\tau} = \vec{\mu} \times \vec{B}$. Explain.

B) Define the direction of the magnetic dipole moment vector with respect to the magnetic field in the center of the loop, so that the above definition is valid. Explain.

C) What are the SI units of magnetic dipole moment?

D) If the current loop has N turns instead of 1, how would you redefine the magnitude of the magnetic dipole moment?

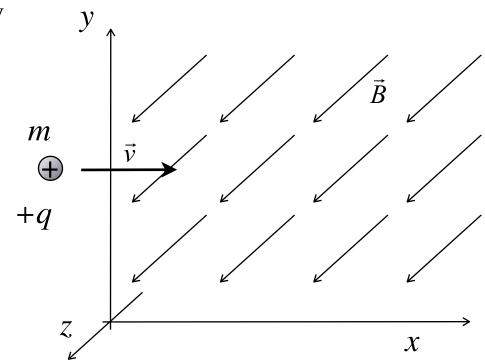
E) A length L of wire carries a current i. It is to be formed into a circular coil with N turns and placed with its dipole moment perpendicular to a magnetic field with magnitude B.
  i) Show that the maximum torque is obtained when $N = 1$
  ii) Find the magnitude of the maximum torque.

F) The magnetic dipole moment of the earth has a numeric value of about 8×10^{22} in SI units. Assume that this is the result of charge flowing in a circle of radius 3,500 km, in the earth's molten outer core. Calculate the current associated with this flowing charge.

AP Physics C
Unit 14 Practice Test   Name ____________________

Part I
Multiple Choice

For questions 1 - 3: A particle of charge $+q$ and mass m is projected with velocity $\vec{v}$ parallel to the $+x$-axis into a uniform magnetic field $\vec{B}$, which is directed in the positive z-direction as shown to the right. The particle moves in a semicircle of radius R.



1. Which of the following best indicates the magnitude and the direction of the magnetic force $\vec{F}$ on the charge just after the charge enters the magnetic field?

	Magnitude	Direction
A)	$\dfrac{kQ^2}{R^2}$	$+y$
B)	$\dfrac{kQ^2}{R^2}$	$-y$
C)	QvB	$+z$
D)	QvB	$+y$
E)	QvB	$-y$

2. If the magnetic field strength is increased, which of the following will be true about the radius R?

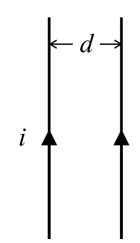
   I R increases if the incident speed is held constant.
   II For R to remain constant, the incident speed must be increased.
   III For R to remain constant, the incident speed must be decreased.

   A) I only   B) II only   C) III only   D) I and II only   E) I and III only

3. Which of the following best indicates the magnitude and the direction of the electric field $\vec{E}$ that could be created so that the charge continues in a straight line after it enters the magnetic field?

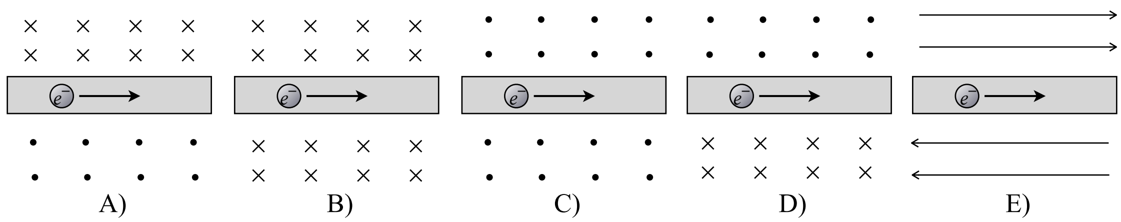
	Magnitude	Direction
A)	$\dfrac{kQ}{R^2}$	$+y$
B)	$\dfrac{kQ}{R^2}$	$-y$
C)	vB	$+z$
D)	vB	$+y$
E)	vB	$-y$

4. Two long parallel wires, separated by a distance d, carry equal currents i toward the top of the page, as shown to the right. The magnetic field due to the wires at a point halfway between them is



   A) zero in magnitude
   B) directed into the page
   C) directed out of the page
   D) directed to the right
   E) directed to the left

5. The diagram below that best represents the magnetic field around a straight wire in which electrons are flowing to the right is



6. In a mass spectrometer, the strength of the magnetic field is 0.1 Tesla. Upon entering the chamber of the spectrometer, a positive ion traveling at 2.0×10^{6} meters per second perpendicular to the magnetic field experiences a magnetic force having a magnitude of 3.2×10^{-14} Newtons. The charge on this positive ion is

A) 6.4×10^{-21} C

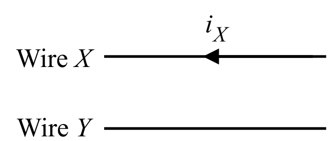
B) 1.6×10^{-19} C

C) 6.4×10^{-9} C

D) 1.6×10^{-9} C

E) 1.6×10^{-21} C

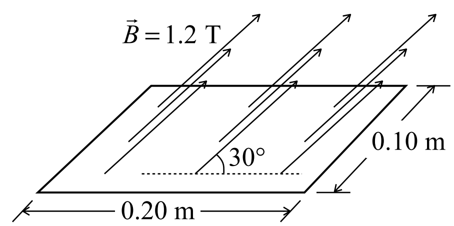
7. Two long, straight parallel wires are held fixed, as shown to the right. There is a known current i_X in wire *X* as shown, and wire *X* experiences a magnetic force of magnitude F_B toward wire *Y*. Which of the following could be true of wire *Y*?



I It carries a current in the same direction as i_X.
II It experiences a force directed away from wire *X*.
III It experiences a force of different magnitude than the force on wire *X*.

A) None B) I only C) II only D) III only E) I or II

8. A uniform magnetic field $\vec{B}$ of magnitude 1.2 T passes through a rectangular loop of wire, which measures 0.10 m by 0.20 m. The field is oriented 30° with respect to the plane of the loop, as shown to the right. What is the magnetic flux through the loop?

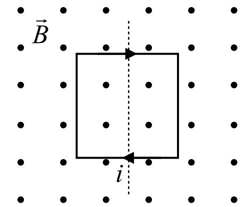


A) Zero
B) 0.012 T•m^2
C) 0.02 T•m^2
D) 0.024 T•m^2
E) 0.048 T•m^2

9. A positively charged particle in a uniform magnetic field is moving in a circular path of radius *r* perpendicular to the field. How much work does the magnetic force *F* do on the charge for half a revolution?

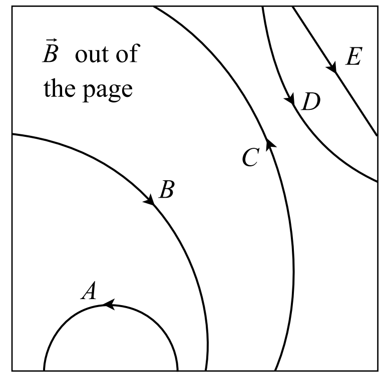
A) $\pi r^2 F$
B) $2\pi rF$
C) πrF
D) $2rF$
E) Zero

10. A loop of wire carrying a steady current i is initially at rest perpendicular to a uniform magnetic field of magnitude B, as shown to the right. The loop is then rotated about the axis shown at a constant rate. The torque on the loop is maximum when the loop has rotated, with respect to its initial position, through an angle of



A) 30°
B) 45°
C) 90°
D) 180°
E) 360°

11. The figure to the right shows the paths of five particles as they pass through the region inside the box that contains a uniform magnetic field $\vec{B}$ directed out of the page. Which particle has a positive charge?



A) A
B) B
C) C
D) D
E) E

12. The units of magnetic dipole moment are

A) Ampere•meter
B) Ampere•meter^2
C) Ampere/meter
D) Ampere/meter^2
E) Ampere

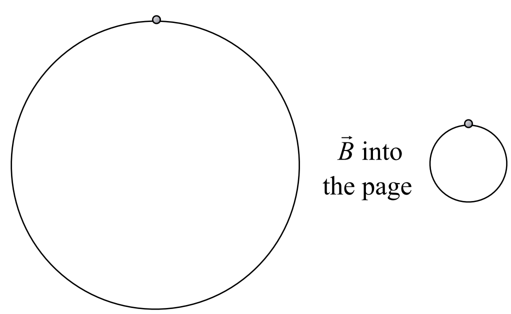
13. A cyclotron operates with a given magnetic field at a given frequency. If the radius of the cyclotron is R, the maximum particle energy is proportional to

A) $1/R$
B) R
C) R^2
D) R^3
E) R^4

14. A charged particle is projected into a region of uniform, parallel $\vec{E}$ and $\vec{B}$ fields. The force on the particle

A) is zero
B) is parallel to the field lines
C) is perpendicular to the field lines
D) is nonzero, but neither parallel nor perpendicular to the field lines
E) is nonzero, and can either be parallel or perpendicular to the field lines

15. An electron and a proton each travel with equal speeds around circular orbits in the same uniform magnetic field, as shown to the right (not to scale). The magnetic field is into the page. Because the electron is less massive than the proton, and because the electron is negatively charged and the proton is positively charged,



A) the electron travels clockwise around the smaller circle and the proton travels counterclockwise around the larger circle
B) the electron travels counterclockwise around the smaller circle and the proton travels clockwise around the larger circle
C) the electron travels clockwise around the larger circle and the proton travels counterclockwise around the smaller circle
D) the electron travels counterclockwise around the larger circle and the proton travels clockwise around the smaller circle
E) the electron travels counterclockwise around the smaller circle and the proton travels counterclockwise around the larger circle

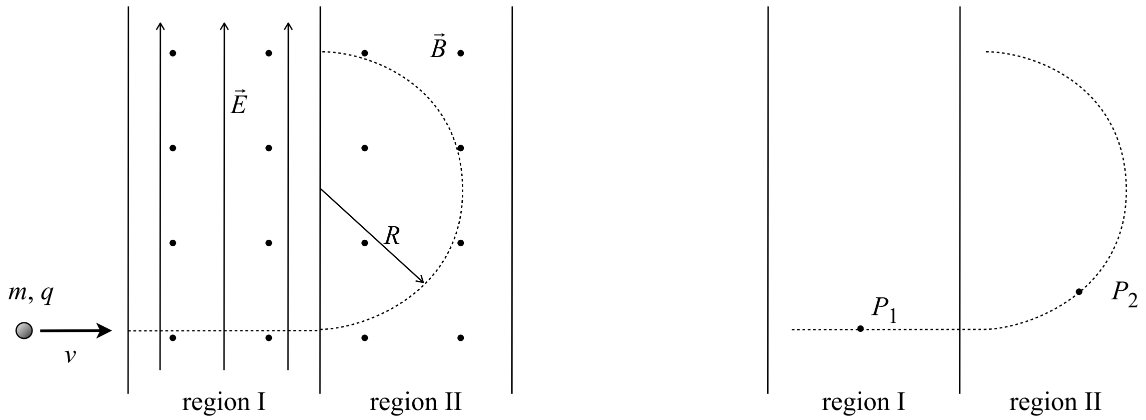
1.E 2.B 3.D 4.A 5.A 6.B 7.B 8.B 9.E 10.C 11.B 12.B 13.C 14.D 15.A

Part II

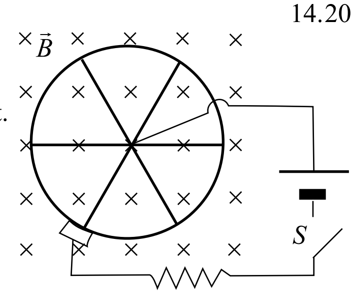
Show your work

Credit depends on the quality and clarity of your explanations

1. An ion of mass m and charge of known magnitude q is observed to move in a straight line through a region of space in which a uniform magnetic field $\vec{B}$ points out of the paper and a uniform electric field $\vec{E}$ points toward the top edge of the paper, as shown in region I below, left. The particle travels into region II in which the same magnetic field is present, but the electric field is zero. In region II the ion moves in a circular path of radius R as shown. (Neglect gravitational forces.)



A) Indicate and label on the diagram above, right, each of the forces on the ion at points P_1 and P_2.
B) Is the ion positively or negatively charged? Explain briefly the reasoning on which you base your conclusion.
C) Derive an expression for the ion's speed v at point P_1 in terms of E and B.
D) Starting with Newton's second law, derive an expression for the mass m of the ion in terms of B, E, q, and R.
E) An ion with the same mass but twice the charge enters region I with the same velocity. Is it possible to adjust the magnitude of either or both fields so that the particle follows the same path? If so, show what adjustment would work; if not, explain why not.



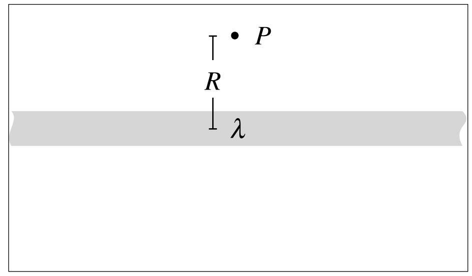
2. A wheel with six spokes is positioned perpendicular to a uniform magnetic field $\vec{B}$ of magnitude 0.5 T. The radius of the wheel is 0.2 meters. The field is directed into the plane of the paper and is present over the entire region of the wheel as shown to the right. When the switch *S* is closed, there is an initial current of 6 A between the axle and the rim, and the wheel begins to rotate. The resistance of the spokes and the rim may be neglected.

   A) What is the direction of rotation of the wheel? Explain.
   B) A short segment of one of the spokes of length *dr* is a distance *r* from the axle of the wheel.
      i) In terms of *r* and *dr*, express the differential torque $d\tau$ about the axle due to this segment.
      ii) Determine by integration the initial torque on the wheel due to all the spokes.
      iii) Is there a net force on the wheel due to the current in the spokes? Explain.
   C) Current returns to the battery through a contact point called a *brush*. The current takes the shortest path from each spoke along the rim to the brush. As the wheel begins to rotate,
      i) is there a torque about the axle due to the current in the rim? Explain.
      ii) is there a net force on the wheel due to the current in the rim? Explain.

AP Physics C
Unit 15 Name_______________________________

Magnetic Field Due to Current

So far our study of the magnetic field has centered on the *effect* of an existing field $\vec{B}$ on a moving charge $q\vec{v}$, and on a current segment $i d\vec{l}$. We now turn our attention to the *cause* of the magnetic field. The same things that feel the effects of magnetic fields are the things that cause them: moving charges and current segments.



Consider an infinite line of positive charge density λ. P is a point a distance R from the line.

A) On the diagram to the right, choose a Gaussian surface that will allow you to find the electric field at P due to the line of charge. Choose parameters that will specify its location and size.

B) Use Gauss' Law to find the magnitude of the electric field E_o at P. Leave your expression in terms of ε_o, not k.

Now imagine that the line of charge is moving with a speed v (which could be the drift velocity of the charge carriers), and there is a point charge at P with charge $+q$ moving at the same speed the other way. Due to length contraction it can be shown that the charge density increases by the factor $\frac{v^2}{c^2}$ in the frame of the moving charge, so it would see a charge density of $\lambda\left(1+\frac{v^2}{c^2}\right)$

C) Write the expression for the electric field E in the moving charge's frame, in terms of the original field E_o:

D) Write the expression for the electric force F the moving charge would feel due to this field:

In our frame, we would see the electric force $E_o q$, and an "extra" force, which we would see as the magnetic force Bqv.

E) Equate the "extra" force to the magnetic force and solve for B plugging in the expression for E_o above:

The moving line of charge represents a current.

F) What is the current i due to the moving charge? (Hint: think of the units):

G) Express the magnetic field B in terms of the current:

Since $\varepsilon_o = \dfrac{1}{4\pi k}$, we can write $\dfrac{1}{\varepsilon_o c^2}$ as $\dfrac{4\pi k}{c^2}$.

H) In SI units, what is the numeric value of $\dfrac{k}{c^2}$ (you shouldn't need a calculator)?

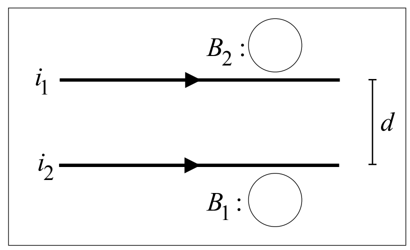
I) What are the SI units of $\dfrac{k}{c^2}$? Show that these units are equivalent to $\dfrac{\mathrm{T \bullet m}}{\mathrm{A}}$:

We abbreviate the constant $\dfrac{1}{\varepsilon_o c^2} = \dfrac{4\pi k}{c^2}$ and call it μ_o.

J) Write the expression for the magnetic field of a long straight current i at a perpendicular distance r from the current, using the constant μ_o: $B =$ (Later we will see an easier way to derive this.) The denominator should remind you that the field lines are circles around the wire.

Examples

1. Two parallel wires a distance d apart carry different currents in the same direction as shown in the diagram.



A) What is the direction of the field B_2, due to the bottom wire, at the location of the top wire? (Indicate its direction in the circle.)

B) What is the direction of the force on the top wire due to its current and the field B_2?

C) What is the direction of the field B_1, due to the top wire, at the location of the bottom wire? (Indicate its direction in the circle.)

D) What is the direction of the force on the bottom wire due to its current and the field B_1?

E) Write an expression for the magnitude of the magnetic field B_2 at the top wire: $B_2 =$

F) Consider a short segment $i_1 dl$ of the top wire. What is the differential force dF on this segment?

$dF =$

G) Write an expression for the force per unit length on the top wire:

H) What is the force per unit length on the bottom wire? Explain.

I) What changes (if anything) in each of the above if the direction of the current in the top wire is reversed?

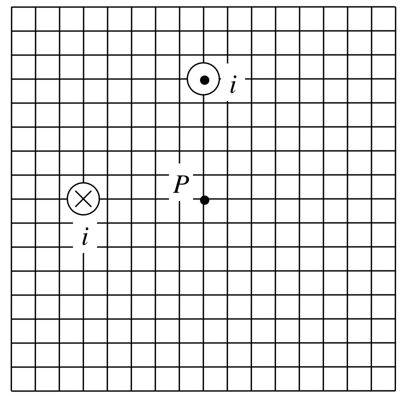
A) ______________________ E) ______________________

B) ______________________ F) ______________________

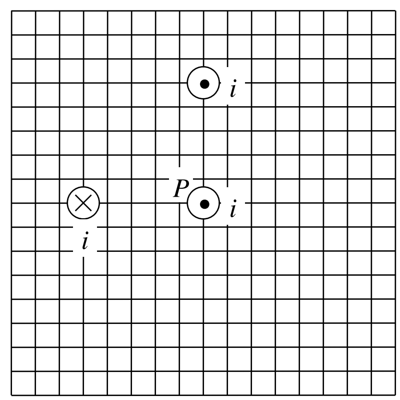
C) ______________________ G) ______________________

D) ______________________ H) ______________________

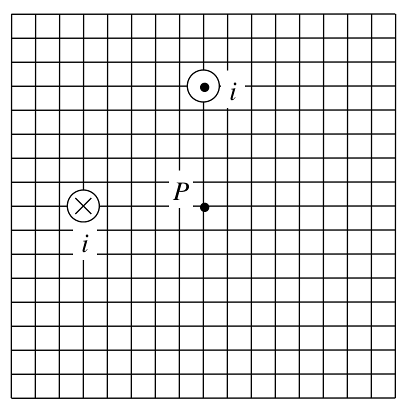
2. Shown to the right are cross-sectional views of two long straight wires that are parallel to each other. One carries current *i* out of the page ($\odot$), and the other carries an equal current *i* into the page ($\otimes$).



A) Draw a vector on the diagram to show the direction of the magnetic field, if any, at point *P*. Explain your reasoning.



B) A third wire, also carrying current *i* out of the page, passes through point *P*. Draw a vector on the diagram to indicate the direction of the magnetic force, if any, exerted on the wire at *P*. Explain your reasoning.

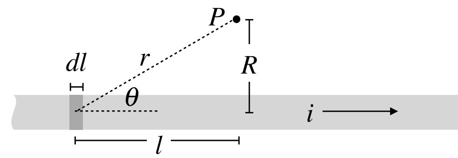


C) The third wire is now moved to a location such that the magnetic field at point P has zero magnitude. Determine its *exact* location and draw the wire on the diagram. Explain your reasoning.

The Biot-Savart Law

We would like to be able to determine the magnetic field due to *any* current-carrying wire, not just straight, infinitely long ones. To do this, we need to break the wire up into short current segments $i d\vec{l}$ and see what contribution dB each segment makes to the total field B.

Consider the current segment shown to the right, part of a long, straight wire carrying current i. If this segment makes a contribution dB to the field at P, then $B = \int dB$ must give the expression you derived above. We make the following "educated guesses" about what dB should look like:



1. dB should be directly proportional to the cause of the field, idl.
2. dB should be directly proportional to $\sin\theta$, because the greatest contribution happens when $\theta = 90°$, and the contribution goes to zero as θ goes to zero or 180°.
3. dB should be proportional to $\frac{1}{r^2}$, so that when we integrate, the total will be proportional to $\frac{1}{R}$

So our candidate for dB is: $dB = K\frac{idl\sin\theta}{r^2}$ where K is yet to be determined. We can integrate with respect to θ, where R is constant.

A) Express l and r in terms of R and θ:

B) Express dl in terms of R and θ:

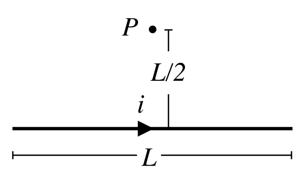
C) Substitute the expressions for dl and r and integrate dB with respect to θ, using appropriate limits:

D) To match the expression on the previous page, what must the constant K be?

We now have the tool we need to find the magnetic field of any wire. This is known as the *Biot-Savart Law* (after the French physicists Jean-Baptiste Biot and Felix Savart — the t's are silent, so it sounds like "Bio-Savar".) The direction of each contribution $d\vec{B}$ to the total field is given by Right-Hand Rule #2.

Examples

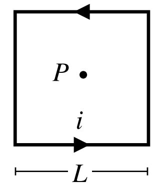
1. A finite wire of length L carries a current i as shown to the right. Point P is a distance $L/2$ from the center of the wire.



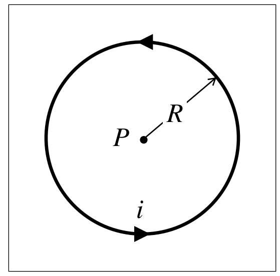
A) What is the direction of the magnetic field $\vec{B}$ at P due to the wire?

B) Modify the integration from the previous page to find the magnitude of $\vec{B}$:

C) Use your result to find an expression for the magnetic field in the center of a square loop of side L carrying a current i:



2. A circular loop of wire of radius R carries current i as shown to the right.
   A) On the diagram, choose and label segment idl.
   B) What is the direction of the contribution $d\vec{B}$ to the total field at P, the center of the loop?



   C) Write the expression for the magnitude of $d\vec{B}$ according to the Biot-Savart Law:

   D) What is the value of the angle θ?

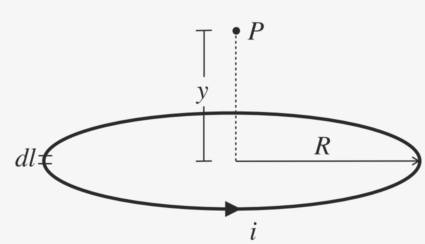
   E) Integrate to find the magnitude of the magnetic field at the center of the loop.

   F) Given a length L of wire, would you get a stronger field in the center by forming it into a circle or a square? Show your reasoning.

Now consider point P a distance y from the center of the loop along the axis of the loop. A segment dl has been labeled on the diagram.

G) On the diagram, draw a vector $d\vec{B}$ representing the direction of the contribution of this segment to the field at P by Right-Hand Rule #2.

H) Write the expression for the magnitude of $d\vec{B}$ according to the Biot-Savart Law:



I) What is the value of the angle θ in the Biot-Savart Law?

J) What is the direction of the net magnetic field $\vec{B}$ at P? Explain.

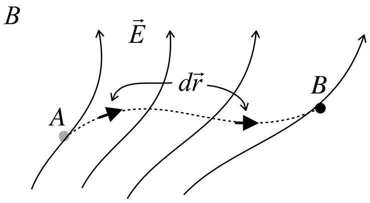
K) On the diagram, draw and label the component of $d\vec{B}$ that points in the direction of the net magnetic field.

L) Integrate to find the total field $\vec{B}$ at P due to the loop.

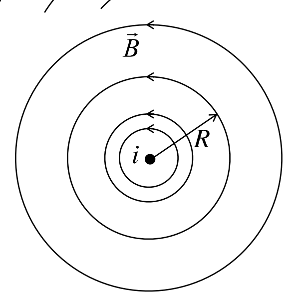
M) Show that your expression reduces to the expression in part E) when $y = 0$.

Ampere's Law

Using the Biot-Savart Law we can determine the magnetic field due to *any* currents by summing the contributions $d\vec{B}$ from small current segments idl. This is similar to finding the electric field $\vec{E}$ due to any charge distribution by summing the contributions $d\vec{E}$ from small charges dq. If we have certain kinds of symmetry, Gauss' Law provides a simpler way to find $\vec{E}$. We will see that, if the magnetic field has certain kinds of symmetry, there will be a simpler way to find $\vec{B}$.



Recall from Unit 10 the definition of electric potential difference between two points A and B in an electric field: Choose an arbitrary path from A to B and evaluate the integral $\Delta V_{AB} = \int_A^B \vec{E} \bullet d\vec{r}$ along the path. This type of integral is called a *line integral*. In Gauss' Law we evaluate a *surface integral*.



Consider the diagram to the right showing a current out of the page, and some of the magnetic field lines due to that current.

A) Write the expression for the magnitude of $\vec{B}$ at a distance R from the wire.

$B =$

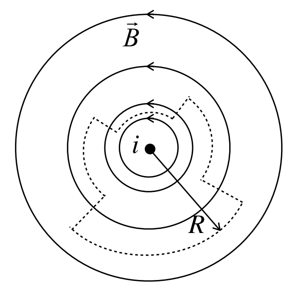
B) Evaluate the line integral $\oint \vec{B} \bullet d\vec{r}$ counterclockwise along a circular path of radius R.

The circle on the integral sign indicates that the path of the line integral is a closed loop.

C) If you choose a circular path with a larger radius, would the line integral be *greater*, *less*, or *the same* as above? Explain.

D) If you integrate along the same path clockwise, what is the value of the integral? Explain.

Consider the path shown to the right, consisting of circular arcs of various radii joined by radial segments. We are going to integrate counterclockwise along this path.



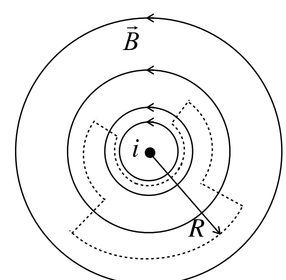
E) When $d\vec{r}$ points along a radial segment, either inward or outward, what is the value of $\int \vec{B} \cdot d\vec{r}$ for that segment? Explain.

F) Let one of the arcs have radius R and subtend an angle θ. What is the length of that segment?

G) What is the value of $\int \vec{B} \cdot d\vec{r}$ for that segment?

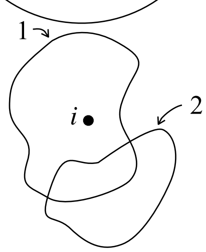
H) What is the value of $\oint \vec{B} \cdot d\vec{r}$ for the entire closed loop?

Now consider the path shown to the right, which doesn't "surround" the current.



I) What is the value of $\oint \vec{B} \cdot d\vec{r}$ counterclockwise around this loop? Explain.

Finally, consider two arbitrarily-shaped loops 1 and 2 shown to the right. Loop 1 "surrounds" the current, and loop 2 doesn't. We can consider each loop to consist of infinitesimal arc-shaped and radial sections.



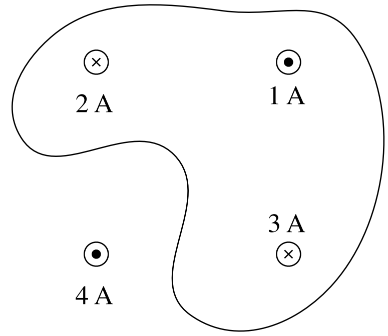
J) What is the value of $\oint \vec{B} \cdot d\vec{r}$ when we integrate:

counterclockwise around loop 1?		clockwise around loop 1?	
counterclockwise around loop 2?		clockwise around loop 2?	

We can summarize this discussion by stating *Ampere's Law*: For any closed loop (called an *Amperian loop*, the counterpart of a Gaussian surface), $\oint \vec{B} \cdot d\vec{r} = \mu_o i_{in}$, where i_{in} is the net current *through* the loop. Since the direction of integration is arbitrary, we choose to define the positive direction by a Right-Hand Rule similar to RHR #2: When you curl the fingers of your right hand in the direction of integration, your thumb points in the direction of positive current. The quantity $\oint \vec{B} \cdot d\vec{r}$ is called the *magnetic circulation*.

Examples

1. Four wires carry currents perpendicular to the page as shown to the right.



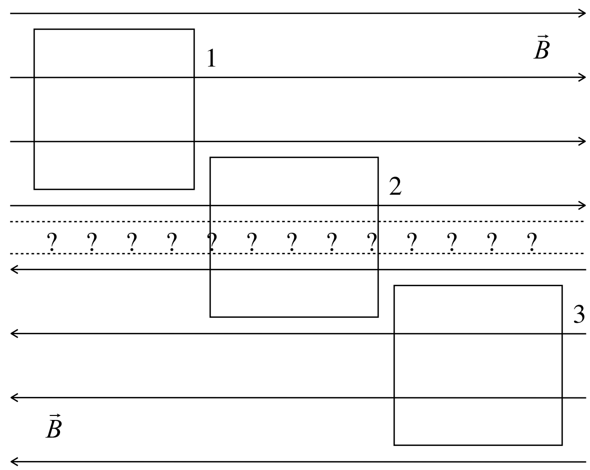
   A) For the Amperian loop shown, what is i_{in}, if the loop is integrated clockwise?

   Counterclockwise?

   B) What is the value of the magnetic circulation around the loop, integrated clockwise?

   Counterclockwise?

2. The drawing to the right shows a region of uniform magnetic field $\vec{B}$ to the right above a pair of dotted lines, and another region of uniform $\vec{B}$ field to the left below the lines. It is unknown what is between the dotted lines.



   A) Three Amperian loops are shown, each a square of side L. For each of the loops, calculate the value of the magnetic circulation $\oint \vec{B} \bullet d\vec{r}$ clockwise around the loop.

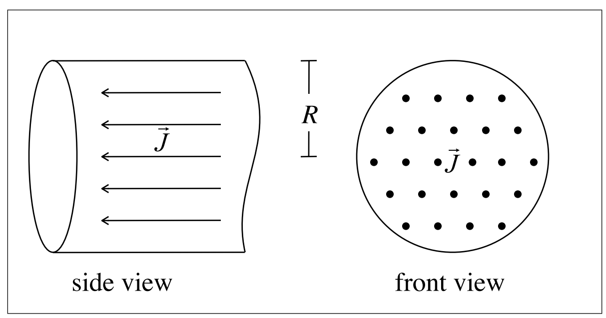
   Loop 1:

   Loop 2:

   Loop 3:

   B) What must exist between the dotted lines? Explain.

3. A solid wire of radius R carries current density $\vec{J}$ uniformly distributed over the interior.



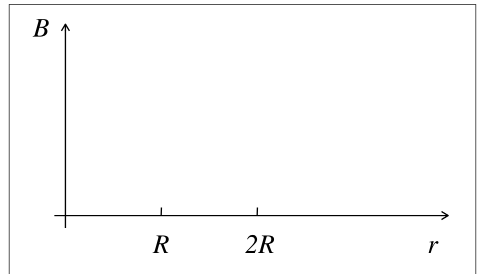
   A) On the front view to the right, draw an Amperian loop of radius $r_1 < R$.

   B) Use Ampere's Law to find the magnitude of $\vec{B}$ at points inside ($r_1 < R$) the wire. State the direction in which you integrate around the Amperian loop.

   C) On the front view to the right, draw an Amperian loop of radius $r_2 > R$.

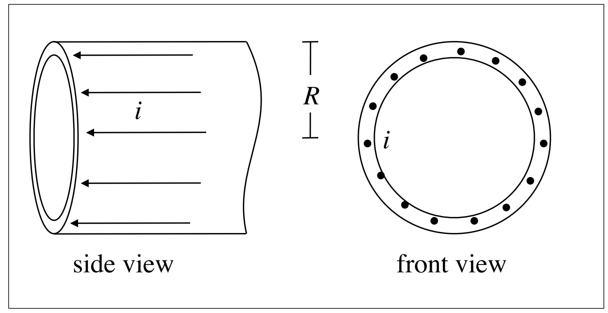
   D) Use Ampere's Law to find the magnitude of $\vec{B}$ at points outside ($r_2 > R$) the wire.

E) On the set of axes to the right, sketch a graph of B vs. r, indicating significant points on the B axis.



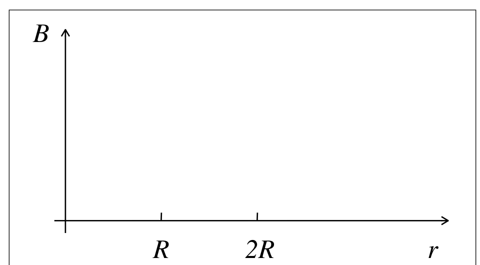
Solid wire

4. A hollow pipe of radius R carries a total current i uniformly distributed over its surface.
   A) On the front view to the right, draw an Amperian loop of radius r_1 < R.
   B) Use Ampere's Law to find the magnitude of $\vec{B}$ at points inside ($r_1 < R$) the pipe.



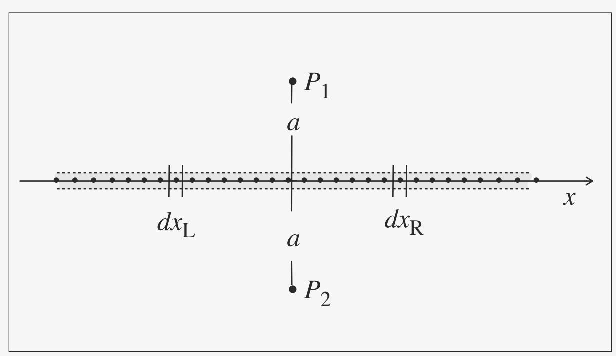
   C) On the front view to the right, draw an Amperian loop of radius r_2 > R.
   D) Use Ampere's Law to find the magnitude of $\vec{B}$ at points outside ($r_2 > R$) the wire.

   E) On the set of axes to the right, sketch a graph of B vs. r, indicating significant points on the B axis.



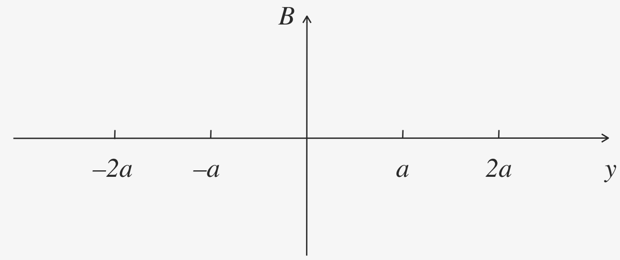
Hollow wire

5. The figure to the right shows a cross-section of an infinite conducting sheet in the x-z plane with a current per unit length λ in the x-direction. The current is flowing in the positive z-direction, and points P_1 and P_2 are a distance a above and below the sheet, respectively. Sections dx_L and dx_R of the sheet are equidistant from points P_1 and P_2
   A) Based on the Biot-Savart Law, indicate and label the contribution $d\vec{B}_L$ to the magnetic field at points P_1 and P_2 due to section dx_L.
   B) Indicate and label the contribution $d\vec{B}_R$ to the magnetic field at points P_1 and P_2 due to section dx_R.
   C) Indicate and label the net magnetic field $\vec{B}_1$ and $\vec{B}_2$ at points P_1 and P_2.
   D) Draw an Amperian loop on the diagram that reflects the symmetry of the magnetic field and will enable you to find the magnitude of the magnetic field $\vec{B}_1$ at P_1.

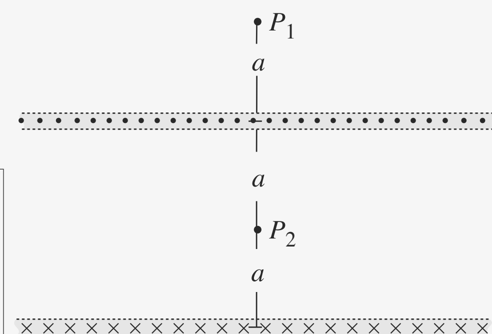


E) Use Ampere's Law and symmetry arguments to find the magnitudes of $\vec{B}_1$ and $\vec{B}_2$.

F) On the axes provided, sketch a graph of the magnitude of the magnetic field above and below the sheet as a function of y. Let the direction of the field at point P_1 be positive, and indicate significant values on the B axis.



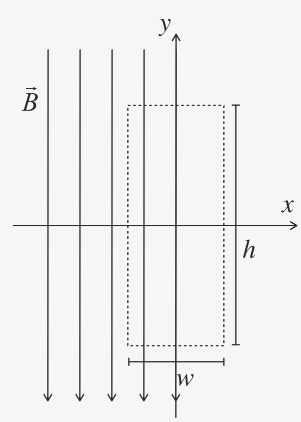
A second identical infinite conducting sheet is placed a distance a below point P_2, with current per unit length λ flowing in the negative z-direction, as shown.



G) Use superposition and symmetry arguments to find the magnitude and direction of the magnetic field $\vec{B}_1$.

H) Use superposition and symmetry arguments to find the magnitude and direction of the magnetic field $\vec{B}_2$.

6. In an idealized situation, a downward magnetic field $\vec{B}$ exists in the region where $x \leq 0$, but drops abruptly to zero for $x > 0$. Consider the Amperian loop in the figure.

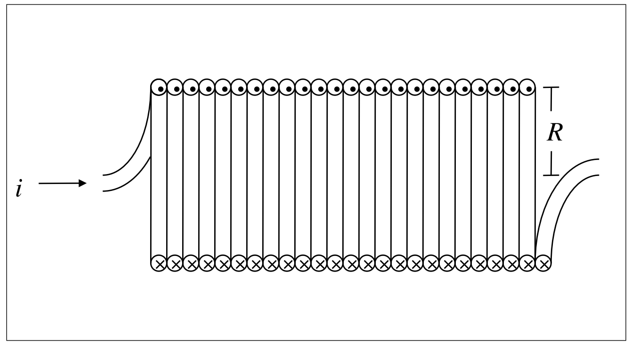


A) Integrate counterclockwise and evaluate $\oint \vec{B} \bullet d\vec{r}$ for the loop.

B) According to Ampere's Law, what must be true of this loop?

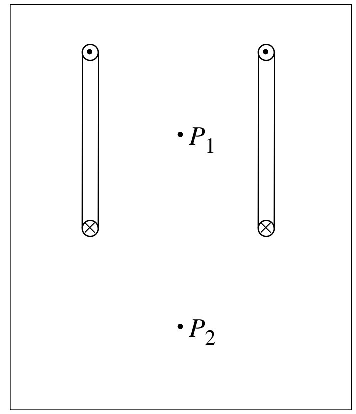
This contradiction shows that magnetic fields don't end abruptly like this.

7. A *solenoid* is a long coil of wire wrapped in the shape of a cylinder. In the figure to the right, the solenoid is split in half showing current coming out of the page at the top and going into the page at the bottom. Consider two of the loops of the solenoid as shown below, right.



A) Indicate the direction of the magnetic field at points P_1 and P_2 due to both loops.

B) How do the strengths of the fields at P_1 and P_2 compare? Explain.

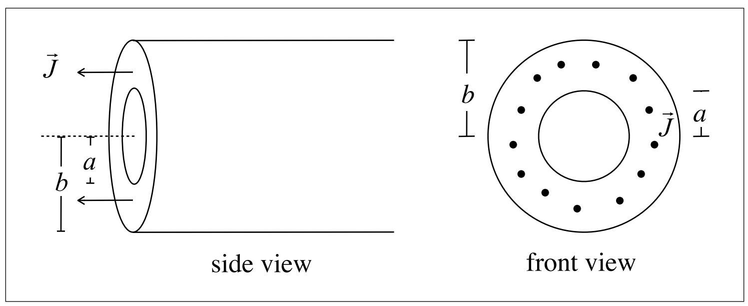


An *ideal solenoid* is one whose length is large compared to R. In this case, the field inside becomes highly uniform, and the field outside is comparatively negligible. Treat the solenoid above as ideal, with n turns per unit length.

C) Choose an Amperian loop that reflects the symmetry of the field, and draw it on the diagram above.

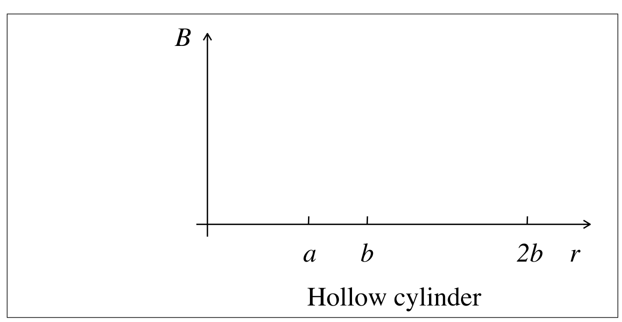
D) Apply Ampere's Law to find the field inside an ideal solenoid. *You should memorize this result.*

8. A long hollow conducting cylinder of inner radius a and outer radius b has a uniform current density $\vec{J}$ distributed over its interior as shown to the right.

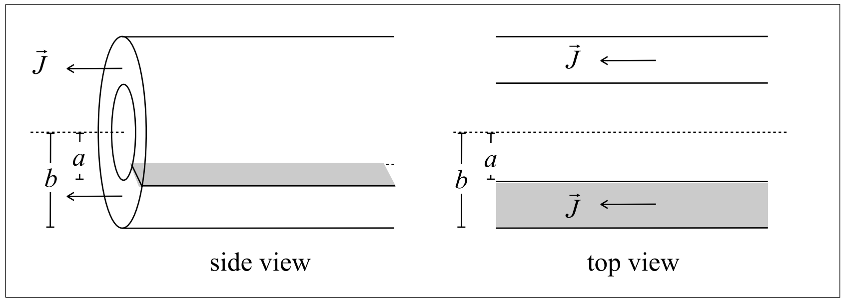


A) Using Ampere's law, derive expressions for the magnetic field intensity at distances r_1 and r_2 from the axis of the cylinder, where $a \le r_1 \le b \le r_2$. Clearly show your Amperian loops on the front view to the right.

B) On the axes to the right, sketch a graph of the magnitude of the magnetic field of the hollow cylinder as a function of r, the distance from the axis of the cylinder. Indicate the values of the field at the specified values of r.



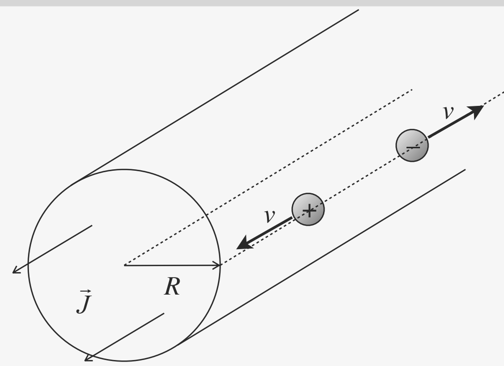
C) Consider a longitudinal cross section of the cylinder as shown in the figure on the left. On the top view, indicate what the magnetic field looks like in that region. Your sketch should reflect the relative strength of the field at different points of the section.



D) Choose a differential area dA for which the magnitude of B is uniform. Sketch dA on the top view diagram, and choose and label parameters to identify the size and location of dA.

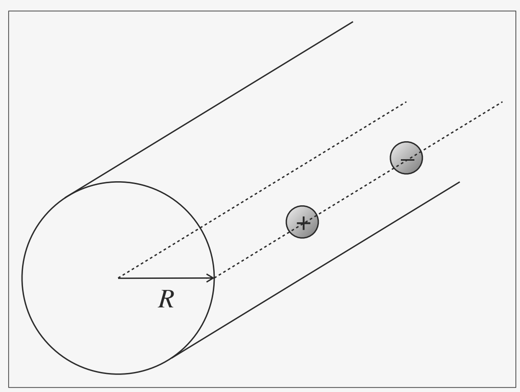
E) Derive an expression for the magnetic flux per unit length Φ / l through the section.

9. A plasma is an ionized gas which contains an equal number of positive charges (ions) and negative charges (electrons). In a current-carrying plasma, the former move in the same direction as the current, and the latter move in the opposite direction. The diagram to the right represents a plasma in the form of a long cylinder of radius R carrying an initially uniform current density $\vec{J}$.



A) On the figure below, draw vectors representing the forces on a positive ion of charge q and an electron of charge $-q$ at the edge of the plasma, assuming that their velocities are v.

B) By using Ampere's Law, derive an expression for the magnitude of the magnetic field at the edge of the plasma.



C) Calculate the force on the ion in terms of the given variables and constants.

D) Describe qualitatively what happens to the plasma. Explain why it is called the "pinch effect."

AP Physics C
Unit 15 Practice Test

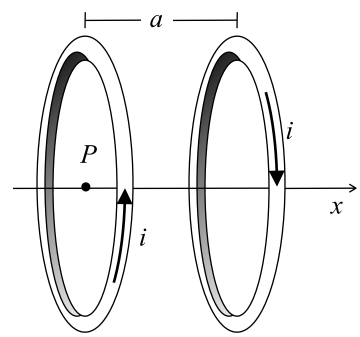
Name ____________________________________

Part I
Multiple Choice

1. Two long, parallel wires, fixed in space, carry currents i_1 and i_2. The force of attraction has magnitude F. What currents will give an attractive force of magnitude $4F$?

   A) $2i_1$ and $(1/2)i_2$
   B) i_1 and $(1/4)i_2$
   C) $(1/2)i_1$ and $(1/2)i_2$
   D) $2i_1$ and $2i_2$
   E) $4i_1$ and $4i_2$

2. Two identical parallel conducting rings have a common axis and are separated by a distance a, as shown to the right. The two rings each carry a current i, but in opposite directions. At point P, the center of the ring on the left, the magnetic field due to these currents is



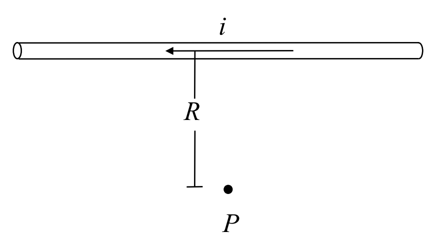
   A) zero
   B) in the plane perpendicular to the x-axis
   C) directed in the negative x-direction
   D) directed in the positive x-direction
   E) none of the above

3. The magnetic field at the center of a solenoid can be calculated using

   I The Biot-Savart Law
   II Ampere's Law
   III Coulomb's Law

   A) I only B) II only C) III only D) I and II E) II and III

4. At point P, a distance R from a long, thin, straight wire carrying current i as shown to the right

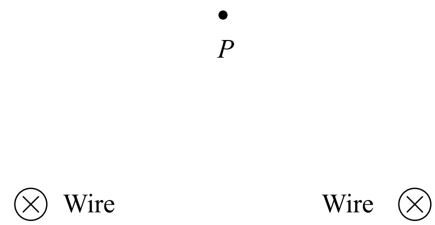


   A) the magnetic field is proportional to $\frac{i}{R}$
   B) the magnetic field is proportional to $\frac{i}{R^2}$
   C) the magnetic field is proportional to iR
   D) the electric potential is proportional to iR
   E) the electric field is proportional to iR^2

5. If the magnetic field $\vec{B}$ is uniform over the area bounded by a square with edge a, the net current through the square is

   A) 0
   B) $4Ba/\mu_o$
   C) Ba^2/μ_o
   D) Ba/μ_o
   E) B/μ_o

6. Two very long parallel wires carry equal currents in the same direction into the page, as shown to the right. At point P, which is equidistant from each wire, the magnetic field is



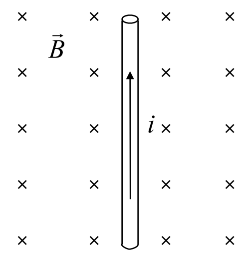
A) zero
B) directed into the page
C) directed out of the page
D) directed to the left
E) directed to the right

7. A hollow copper pipe carries current i parallel to the axis of the pipe. The magnetic field due to the current is

I radially outward outside the pipe
II zero inside the pipe
III circular inside the pipe

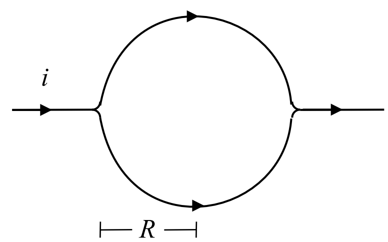
A) I only   B) II only   C) III only   D) I and II   E) II and III

8. A wire carries current i toward the top of the page as shown to the right, in a region where a uniform magnetic field $\vec{B}$ points into the page. The net magnetic field is due to the uniform field and the field of the wire. As a result, the wire will experience



A) a force directed toward the region of stronger net magnetic field
B) a force directed toward the region of weaker net magnetic field
C) a force in the direction of the uniform magnetic field
D) a force in the direction opposite the uniform magnetic field
E) a net torque

9. A straight conductor carrying a current i is split into identical semicircular turns of radius R as shown to the right. The magnetic field at the center of the circular loop so formed is



A) $\mu_0 i/2R$
B) $\mu_0 i/4R$
C) $\mu_0 i/R$
D) $2\mu_0 i/R$
E) zero

For questions 10 - 12: Consider Ampere's Law: $\oint \vec{B} \bullet d\vec{r} = \mu_0 i_{\text{in}}$.

10. The integration must be over any

A) surface
B) closed surface
C) path
D) closed path
E) closed path that surrounds all the current producing $\vec{B}$

11. The symbol $d\vec{r}$ is

A) An infinitesimal piece of the wire that carries current i
B) in the direction of $\vec{B}$
C) perpendicular to $\vec{B}$
D) a vector whose magnitude is the length of the wire that carries i
E) none of these

12. The direction of the integration

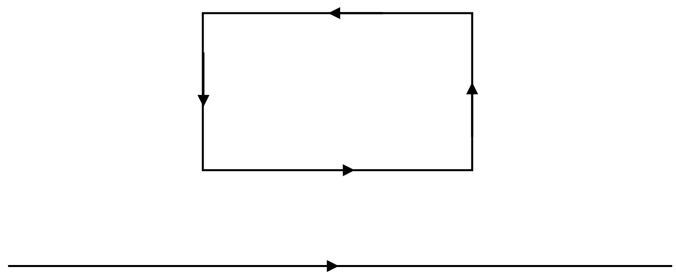
A) must be clockwise
B) must be counterclockwise
C) must be such as to follow the magnetic field lines
D) must be along the wire in the direction of the current
E) none of these

13. A long, straight cylindrical conducting shell carries current *i* parallel to its axis and uniformly distributed over its cross section. The magnitude of the magnetic field is greatest

A) at the inner surface of the shell
B) at the outer surface of the shell
C) inside the shell at the center
D) outside from the shell far from the shell
E) none of these

14. Solenoid 2 has twice the radius and six times the number of turns per unit length as solenoid 1. The ratio of the magnetic field in the interior of solenoid 2 to that of solenoid 1 is

A) 2:1
B) 4:1
C) 6:1
D) 1:1
E) 1:3



15. A long straight wire carries a current to the right as shown above. A rectangular loop of wire is above the long wire, and also has a current flowing counterclockwise as shown. Which of the following statements is true?

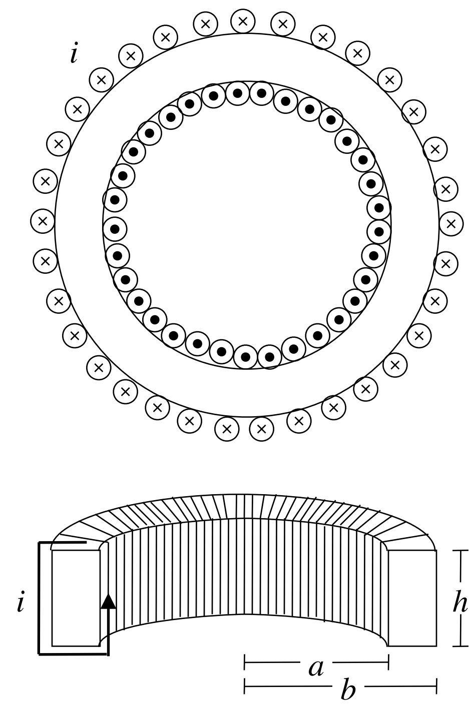
A) The net magnetic force on the loop is upward, and there is also a net torque on the loop.
B) The net magnetic force on the loop is zero, and the net torque on the loop is zero.
C) The net magnetic force on the loop is downward, and there is also a net torque on the loop.
D) The net magnetic force on the loop is zero, but there is a net torque on the loop.
E) The net magnetic force on the loop is downward, and the net torque on the loop is zero.

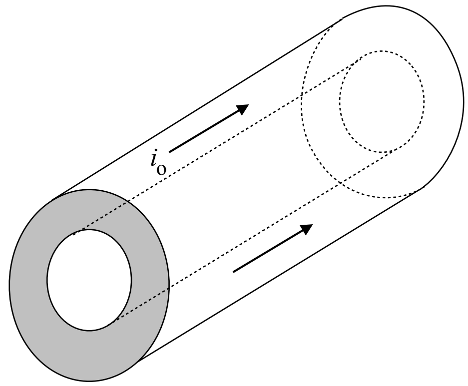
1.D 2.C 3.D 4.A 5.A 6.E 7.B 8.B 9.E 10.D 11.E 12.E 13.B 14.C 15.E

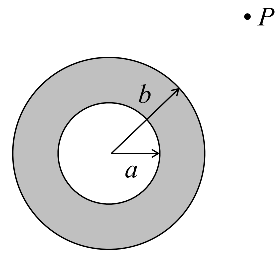
Part II
Show your work
Credit depends on the quality and clarity of your explanations

1. A *toroid* is a solenoid bent into the shape of a doughnut. A *rectangular toroid* is a toroid with a rectangular cross section rather than a circular cross section. Consider the rectangular toroid with inner radius a, outer radius b, and height h as shown in the drawings to the right. The toroid has N turns of wire carrying a current i as shown. As with the ideal solenoid, the magnetic field is entirely confined to the interior of the toroid.

   A) In each drawing, sketch the magnetic field lines within the toroid.
   B) Use Ampere's Law to find an expression for the strength of the magnetic field inside the toroid at a distance r from the center.
   C) Explain how this expression is related to the expression for the magnetic field of an ideal (straight) solenoid, assuming that a and b are almost equal.
   D) Find the magnetic flux through a rectangular cross section of the toroid.







Cross-sectional View
(current into the page)

2. A section of a long conducting cylinder with inner radius a and outer radius b carries a current i_o that has a uniform current density, as shown in the figure above.

   A) Using Ampère's law, derive an expression for the magnitude of the magnetic field in the following regions as a function of the distance r from the central axis.
      i) $r < a$
      ii) $a < r < b$
      iii) $r = 2b$

   B) On the cross-sectional view in the diagram above, indicate the direction of the field at point P, which is at a distance $r = 2b$ from the axis of the cylinder.

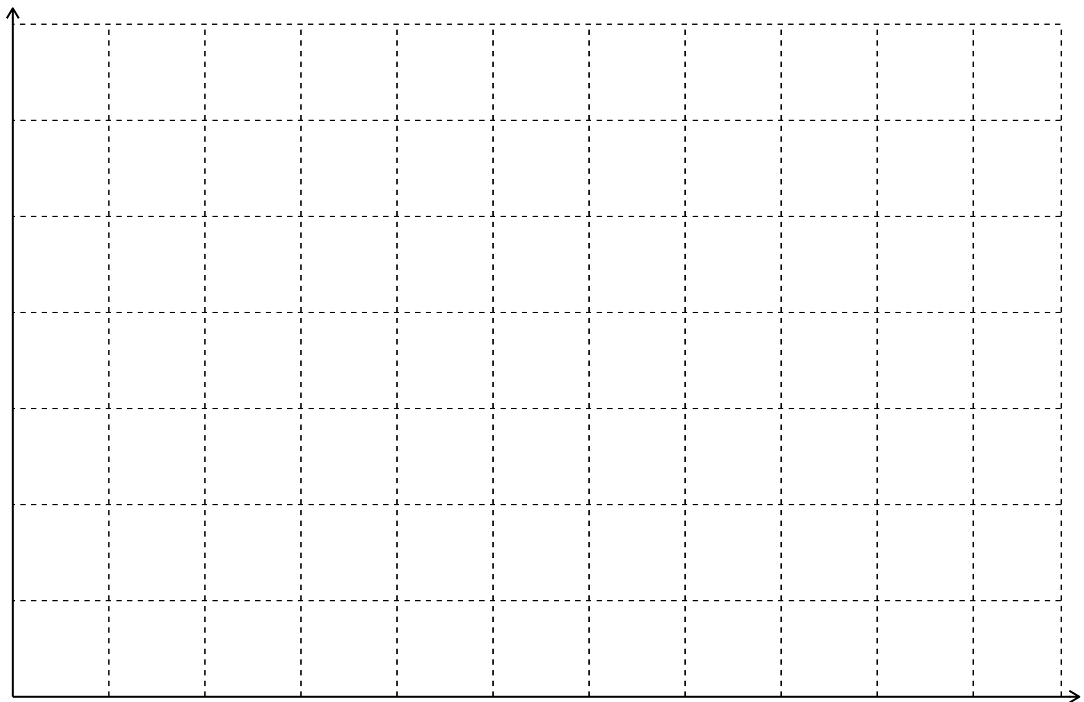
   C) An electron is at rest at point P. Describe any electromagnetic forces acting on the electron. Justify your answer.

(*continued on next page*)

Now consider a long, <u>solid</u> conducting cylinder of radius b carrying a current i_o. The magnitude of the magnetic field inside this cylinder as a function of r is given by $B = \frac{\mu_o i_o r}{2\pi b^2}$. An experiment is conducted using a particular solid cylinder of radius 0.010 m carrying a current of 25 A. The magnetic field inside the cylinder is measured as a function of r, and the data is tabulated below.

Distance r (m)	0.002	0.004	0.006	0.008	0.010
Magnetic Field B (T)	1.2×10^{-4}	2.7×10^{-4}	3.6×10^{-4}	4.7×10^{-4}	6.4×10^{-4}

D) i) On the graph below, plot the data points for the magnetic field B as a function of the distance r, and label the scale on both axes. Draw a straight line that best represents the data.

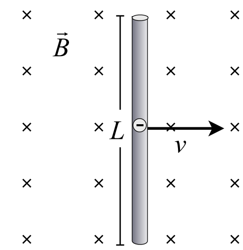


ii) Use the slope of your line to estimate a value of the permeability μ_o.

AP Physics C
Unit 16 Name____

Motional Emf

The emf supplied by a battery in a circuit comes from a conversion of chemical energy to electrical energy. It is also possible to convert mechanical energy into electrical energy. Imagine moving a wire of length L through a magnetic field $\vec{B}$ at a speed v such that both the wire and the velocity are perpendicular to $\vec{B}$ and to each other as shown to the right. One of the free electrons in the wire is shown.



A) What is the direction of the magnetic force on this electron?

The free electrons in the wire will move in response to this force, causing an electric field within the wire.

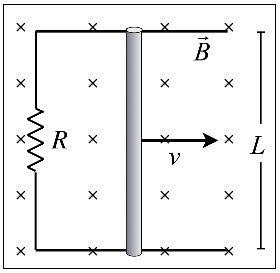
B) What is the direction of this electric field?

The electrons rapidly come to equilibrium where the magnetic force is balanced by the force due to the electric field.

C) What is the magnitude of the electric field once equilibrium is established?

D) What is the potential difference between the ends of the wire once this electric field is established? Explain.

This potential difference is called *motional emf.* We can cause this potential difference to generate a current by connecting it to a complete circuit. For example, we can set up two conducting rails as shown to the right, a distance L apart, and connect the two rails together with a resistor. Ignore the resistance of the wire and rails.



E) What is the current through the resistor? Indicate its direction on the drawing and explain why it goes in this direction.

F) What is the magnitude of the magnetic force on the wire due to this current? Indicate its direction on the drawing and explain why the force is in this direction.

G) What is the mechanical power required to keep the wire moving at this speed?

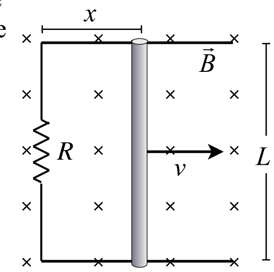
H) What is the rate at which energy is being dissipated in the resistor?

I) Explain the connection between your answers to parts G) and H).

We can think of this system as a *generator*, which converts mechanical energy into electrical energy.

Faraday's Law of Induction and Lenz's Law

We say that the emf in the circuit on the previous page is *induced* (caused) by the motion of the wire through the magnetic field. There is another, more general way to see how emf can be induced. The diagram to the right shows the wire moving along the rails. At the moment shown, the wire is a distance x from the left end of the rails.



A) At this moment, what is the magnetic flux Φ_B through the area bounded by the circuit?

We choose the direction *into the page* as positive, so this flux is positive.

B) What is the *rate of change* of the flux $\frac{d\Phi_B}{dt}$?

C) How does this relate to the emf caused by the moving wire?

D) Is this rate *positive* or *negative* according to our choice of sign?

The induced current causes its own magnetic field.

E) Within the area bounded by the circuit, what is the direction of the magnetic field due to the induced current? Explain.

F) Is the flux due to this induced magnetic field *positive* or *negative* given our choice of sign?

We now move the wire at the same speed, but in the opposite direction, toward the resistor.

G) Is the rate of change of flux within the circuit *positive* or *negative*?

H) What is the direction of the current through the resistor?

I) What is the direction of the induced magnetic field within the area bounded by the circuit?

J) Is the flux due to this induced magnetic field *positive* or *negative* given our choice of sign?

K) In parts D) and F), and parts G) and J), how are the signs of the induced flux and the rate of change of flux related?

In this example, we are changing the flux by changing the area. In general, what other ways could we change the flux of the magnetic field through a loop of wire?

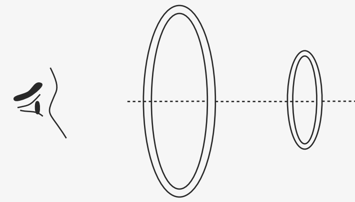
Michael Faraday found that no matter how we change the flux, there will be an induced emf equal to the *rate of change* of flux, and that this induced emf will *oppose* the change that caused it. This is known as *Faraday's Law of Induction*:

$$\varepsilon = -\frac{d\Phi_B}{dt}.$$

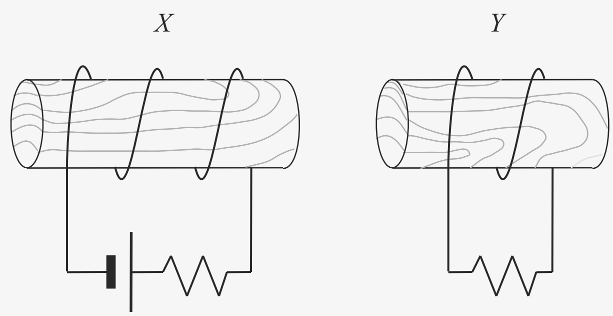
The negative sign indicates that the emf opposes the change in flux, and this aspect of Faraday's Law is known as *Lenz's Law* after Heinrich Lenz, a Russian physicist.

Examples

1. Two conducting loops are arranged along a common axis as shown to the right. An observer sights along the axis from the left. If a clockwise current is increasing in the larger loop, what is the direction of the induced current in the smaller loop? Explain using Lenz's Law.



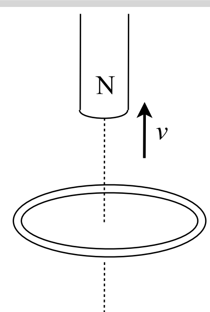
2. Coils X and Y are wound around wooden dowels. Determine the direction of the induced current (left-to-right or right-to-left) in coil Y in the following cases, and explain using Lenz's Law:



   A) Coil Y is moved toward coil X.

   B) The current in coil X is decreased.

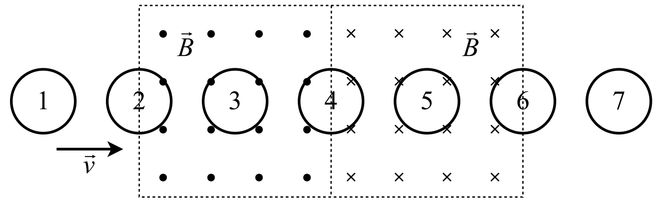
3. The permanent magnet shown to the right is moved along the axis of a circular loop, away from the loop.



   A) What is the direction of the induced current in the loop, as viewed from above? Explain using Lenz's Law.

   B) If, instead, the magnet is held stationary, but the diameter of the circular loop is steadily increased, what is the direction of the induced current in the loop, as viewed from above? Explain using Lenz's Law.

4. A circular loop moves with constant velocity through regions where uniform magnetic fields of the same magnitude are directed out of (on the left) and into (on the right) the page, as indicated in the diagram.



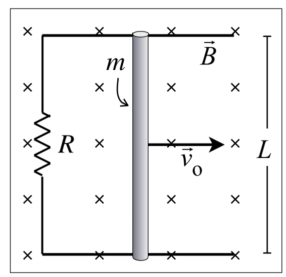
A) For each numbered position of the loop, determine whether the induced current is clockwise (*cw*), counterclockwise (*ccw*), or zero (0).

1:	2:	3:	4:	5:	6:	7:

B) For those numbered positions where the induced current is nonzero, explain your choice of direction using Lenz's Law.

C) Rank the magnitudes of the induced currents where they are nonzero, and explain using Faraday's Law of Induction.

5. The diagram to the right shows the system from page 16.1. The wire has mass m, and it is given an initial speed v_o to the right and released. Neglect friction and gravitational forces.



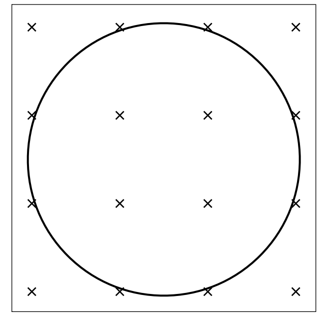
A) Using Newton's second law, write a differential equation in v whose solution gives the speed of the rod as a function of time. Be careful of signs.

B) Integrate the differential equation to obtain the speed of the rod as a function of time.

C) Determine the rate at which heat is dissipated by the resistor R as a function of time.

D) Show that the total energy dissipated by the resistor is equal to the initial kinetic energy of the rod.

6. A circular loop of wire of area A and resistance R is placed in a spatially uniform magnetic field $\vec{B}$ directed into the page and perpendicular to the plane of the loop as shown to the right. The magnetic field is gradually reduced from an initial value of B_o, in such a way that the magnetic field strength as a function of time is $B(t) = B_o e^{-\alpha t}$.



A) Indicate on the diagram the direction of the induced current. Explain your choice using Lenz's Law.

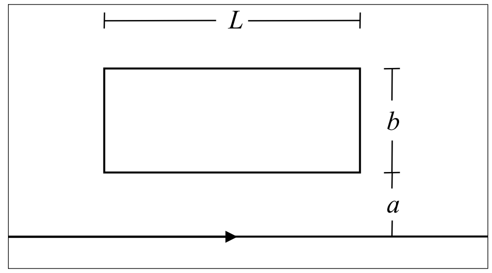
B) Determine an expression for the current induced in the loop as a function of time.

C) Determine an expression in terms of B_o, A and R that describes the total quantity of charge that flows past a point in the loop during the time the magnetic field is reduced from B_o to zero.

D) Show that your answer to part C) is true regardless of how the magnetic field B depends on time.

E) Determine an expression for the amount of energy dissipated as heat in the loop during the time the magnetic field is reduced from B_o to zero, in terms of B_o, A, R and α .

7. As shown to the right, a rectangular loop is located next to a long straight wire carrying a current which varies with time as $i = i_{max}\sin(\omega t)$. The wire and the loop are in the plane of the page and fixed in space. The current in the long wire is in the direction shown in the diagram from $t = 0$ to $t = \pi/\omega$.



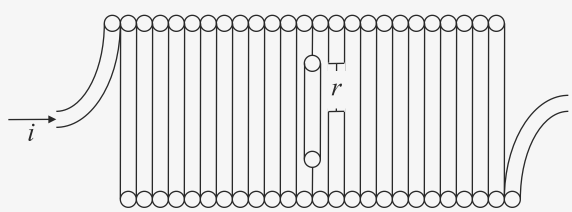
A) On the diagram, draw a differential area dA in the loop for which the magnetic field of the wire is constant. Indicate parameters that specify its size and position.

B) Find the magnetic flux Φ_B through the loop at time t in terms of a, b, L, i_{max}, ω, t, and constants.

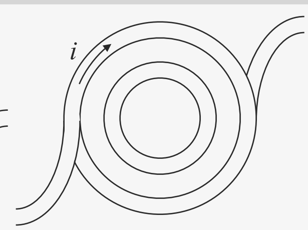
C) Indicate on the diagram the direction of the resulting current that is induced in the loop at time $t = \pi/\omega$.

D) Determine the magnitude of the emf that is induced in the loop at time $t = \pi/\omega$ in terms of the above variables.

8. The long solenoid shown to the right has 1800 turns per meter and carries current i. At its center is placed a single circular loop of wire of radius $r = 2.0$ cm and resistance $R = 5\ \Omega$.



Side view



End view

A) Determine the magnetic flux through the one-turn circular loop in terms of i.
($\mu_o = 4\pi \times 10^{-7}$ Tm/A).

B) If the current in the solenoid is reduced at a steady rate of 0.5 A/s, what is the induced emf in the loop?

C) What is the induced current in the loop?

D) The solenoid current is clockwise as viewed from the left end of the solenoid as shown in the end view. In this view, what is the direction of the magnetic field of the solenoid?

E) According to Lenz's Law, what is the direction of the induced magnetic field in the one-turn loop? Explain.

F) Is the induced current in the one-turn loop *clockwise* or *counterclockwise*?

The solenoid current falls to zero and then reverses direction, now *increasing* at the same rate of 0.5 A/s.

G) What is the induced current in the loop?

H) What is the direction of the induced current in the loop? Explain using Lenz's Law.

The one-turn loop is now replaced with a 5-turn loop, made of the same wire and with the same radius *r*. The solenoid current is varied as above.

I) What is the resistance of the 5-turn loop?

J) What is the emf induced in the 5-turn loop? Explain in terms of the induced emf in each turn of the loop.

K) What is the induced current in the 5-turn loop?

The solenoid current is now varied with time as $i(t) = (1\ \text{A})\sin\left(\dfrac{t}{0.25\ \text{s}}\right)$. This is called *alternating current*, since the direction of the current changes periodically.

L) Determine the magnitude of the induced current in the 5-turn loop as a function of time.

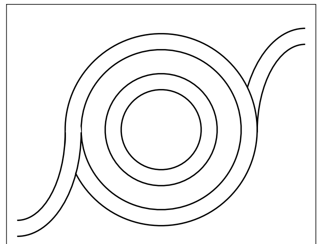
M) How does the *frequency* of the alternating current in the 5-turn loop compare to the *frequency* of the solenoid current?

N) How does the *phase* of the alternating current in the 5-turn loop compare to the *phase* of the solenoid current?

This system is known as a *transformer*. For a given sinusoidal emf in the solenoid (called the *primary coil*), a sinusoidal emf will be generated in the loop (called the s*econdary coil*) with the same frequency, but a different emf, depending on the number of turns in the secondary coil. For example, your laptop charger has a transformer that converts the 110 V in an outlet to about 18 V.

Induced Electric Fields

Consider again the solenoid with a one-turn loop as in example 8 on page 16.6.



A) How does your answer to part B) of example 8 (the induced emf) depend on

i) the radius of the loop?

ii) the resistance of the loop?

The solenoid current changes at a constant rate so that there is a steady clockwise current induced in the loop. For current to flow in the loop there must be an electric field pushing the charges. This is called the *induced electric field*.

B) On the end view above, draw an electric field line in the loop to show its direction.

C) What is the magnitude of the electric field responsible for the induced current?

D) How does the magnitude of the induced electric field depend on

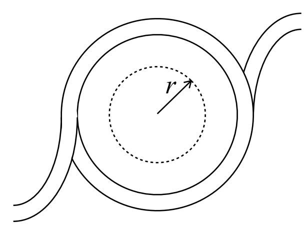
i) the radius of the loop?

ii) the resistance of the loop?

E) If the loop were replaced by a loop with *infinite* resistance, how would that affect the induced electric field?

F) If the loop were replaced by *empty space*, how would that affect the induced electric field?

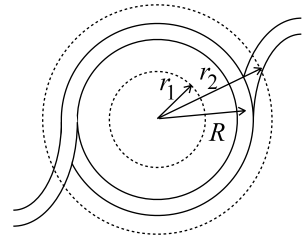
G) The diagram to the right shows the solenoid with the loop removed. A circular path of radius $r = 2$ cm is shown. Consider the path integral $\oint \vec{E} \bullet d\vec{s}$ evaluated around this circular path. What is the value of this path integral?



H) How is the value of this path integral related to the rate of change of magnetic flux within the path?

This relationship is true in general, for any closed path in a changing magnetic field. Using the path integral notation, write a generalized expression for Faraday's Law of induction:

Inductance



Now consider the general case of a solenoid of radius *R*, number of turns per unit length *n*. The solenoid current changes at an arbitrary rate $\frac{di}{dt}$. The diagram to the right shows the solenoid with two circular paths drawn. The inner path has a radius $r_1 < R$, and the outer one has a radius $r_2 > R$.

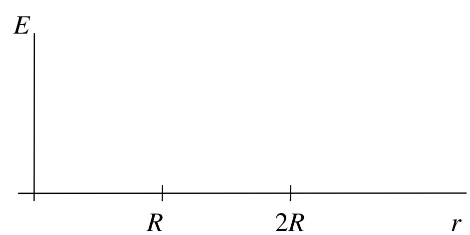
A) What is the induced emf around the inner loop?

B) What is the magnitude of the induced electric field around the inner loop?

C) What is the induced emf around the outer loop?

D) What is the magnitude of the induced electric field around the outer loop?

E) Determine the values of *E* at distances of *R* and 2*R*.



F) On the axes to the right, sketch a graph of the magnitude of the induced electric field as a function of *r*, the distance from the center of the solenoid.

Note that there are two contributions to the net electric field at *R*, within the wire of the solenoid. One is the electric field responsible for the solenoid current *i*, which is provided by some external emf, and the other is the induced electric field due to the changing flux.

G) Given the table below, determine the direction of the induced electric field (*cw*, *ccw*, or 0) in each case, by applying Lenz's Law:

i	cw	cw	cw	ccw	ccw	ccw	0	0	0
$\lvert i \rvert$	de-creasing	constant	in-creasing	de-creasing	constant	in-creasing	in-creasing cw	constant	in-creasing ccw
Induced *E*									

H) The induced electric field causes an induced emf in *each turn* of the solenoid. Make a general statement about how the induced emf affects the solenoid current.

The induced emf is often called the "back emf" for this reason.

I) If the length of the solenoid is ℓ, how many turns are there?

J) Express the total induced emf in the solenoid in terms of $\dfrac{di}{dt}$:

The proportionality constant in part J) involves the structure of the solenoid. We call this constant the *inductance* of the solenoid, symbolized by L.

K) What are the SI units of inductance?

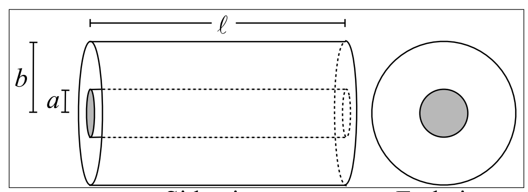
This unit is called the *Henry* (H) after the American physicist Joseph Henry. (The good news: This is your last unit!)

In general, a system where a change in current results in an emf is called an *inductor*. The inductance of an inductor is similar in many ways to the capacitance of a capacitor. The voltage across a capacitor V_C is proportional to the charge q on its plates, and the voltage (emf) of an inductor ε_L is proportional to the rate of change of current *di/dt* through it. Capacitors store energy in the electric field, and inductors store energy in the magnetic field.

Calculating Inductance

The example of the solenoid can show us how to calculate the inductance of any inductor. Show that for a solenoid, the inductance is given by: $L = \dfrac{(\text{Number of turns})(\text{Flux per turn})}{(\text{current})} = \dfrac{N\Phi_B}{i}$

Example



Side view End view

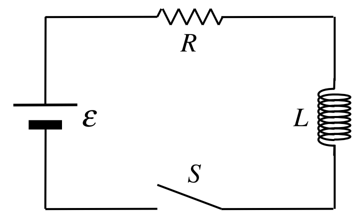
A length ℓ of coaxial cable consists of a wire of radius a inside a concentric conducting shield of radius b, as shown to the right. Current i flows in opposite directions in the wire and the shield.

A) Use Ampere's Law to find the magnitude of the magnetic field between the wire and the shield. Draw your Amperian loop on the end view to the right.

B) Calculate the magnetic flux through the space between the wire and the shield (a rectangle of area $\ell(b-a)$). Draw the differential area dA on the side view, showing parameters that specify its size and location.

C) Calculate the inductance per unit length of the coaxial cable.

Inductors and *LR* Circuits



As with capacitors, when we wish to study the behavior of inductors in a circuit, we care only about the inductance L, not about the geometry of the inductor. The symbol for an inductor in a circuit looks like a coil, as shown in the circuit to the right, which is called a *simple LR circuit* for obvious reasons. When the switch S is closed, the emf tries to send current clockwise around the circuit, but the inductor L tries to oppose that change with an emf $\varepsilon_L = -L\dfrac{di}{dt}$ of its own in the other direction.

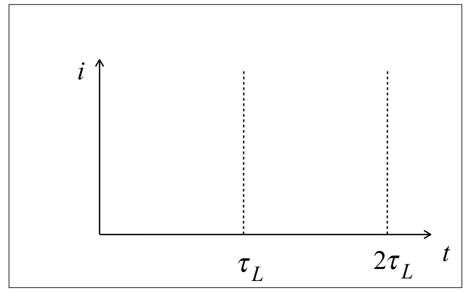
A) Apply the loop theorem to the circuit to find an equation relating the three circuit elements once the switch is closed. Your equation should be a differential equation in i.

B) Separate variables and integrate the differential equation to find an expression for the current in the *LR* circuit as a function of time.

C) What is the "inductive time constant" τ_L for the *LR* circuit? Show that this quantity has units of time.

D) By analogy with the capacitor, how does an inductor behave "initially" and "after a long time?"

E) On the axes to the right, sketch a graph of the current in the LR circuit as a function of time. Indicate significant point on the i axis.

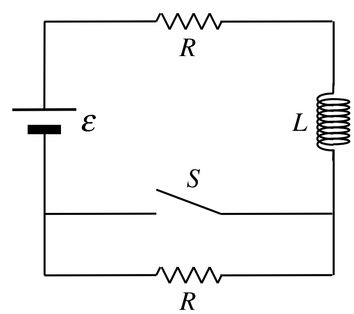


F) Multiply each term in the differential equation from part A) by i. List each term separately, and describe what each term represents.

G) One of the terms above involves the inductance L. Use this term to find by direct integration the total energy U_L stored in the inductor when it carries a current i.

Example

When the switch S in the circuit shown to the right is closed, an inductance L is in series with a resistance R and a battery of emf ε .

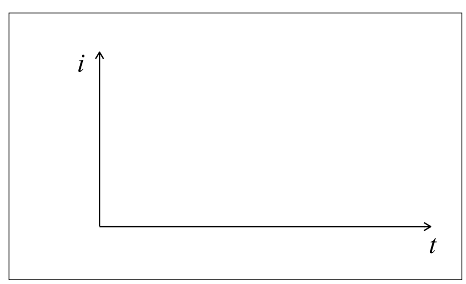


A) Determine the current i_1 in the circuit after the switch S has been closed for a very long time. Explain your reasoning.

After being closed for a long time, the switch S is opened at time $t = 0$. The questions that follow pertain to the time after this ($t > 0$).

B) Determine the current i_2 in the circuit after the switch has been opened for a very long time. Explain your reasoning.

C) On the axes to the right, sketch a graph of the current as a function of time t and indicate the values of the currents i_1 and i_2 on the vertical axis.



D) By relating potential differences and emf's around the circuit, write a differential equation that can be used to determine the current as a function of time.

E) Integrate to determine the equation for the current as a function of time for $t > 0$.

Magnetic Energy Density

When we studied parallel plate capacitors, we divided the energy stored in the capacitor $\left(U_C = \frac{1}{2}\frac{1}{C}Q^2\right)$ by the volume between the plates to find the electric energy density $u_E = \frac{1}{2}\varepsilon_o E^2$.

This is a general result for any electric field, whether or not it is between two capacitor plates. In a similar way we can find the energy density associated with a *magnetic* field. Our tool is the solenoid instead of the capacitor.

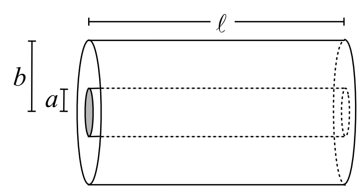
On page 16.10 you found the inductance of a solenoid of radius R, length ℓ and number of turns per unit length n.

A) If the solenoid current is i, how much energy U_L is stored in the solenoid?

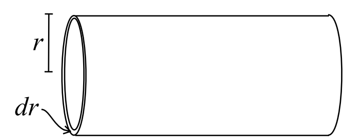
B) Divide the above expression by the volume of the solenoid to find the energy density u_B in terms of B.

Examples

1. Consider the coaxial cable from page 16.10. You used Ampere's Law to find an expression for the magnetic field in the space between the wire and the shield.



A) Use the expression above for energy density to find the energy density as a function of r between the wire and the shield.



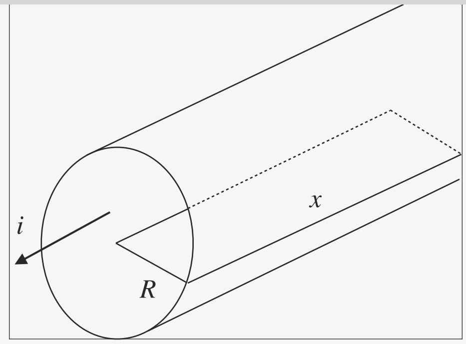
B) Consider a volume element dV consisting of a cylindrical shell of radius r ($a < r < b$), thickness dr, and length ℓ. What is the differential energy dU_B contained in this volume element?

C) Integrate to find the total energy per unit length stored in the cable between the wire and the shield (neglect the energy stored in the wire itself).

D) Calculate the inductance per unit length of the cable, and compare your result to your answer from page 16.10.

2. A long cylindrical wire of radius R carries a current i uniformly distributed over the cross section of the wire.

A) Using Ampere's Law, derive an expression for the magnitude of the magnetic field B inside the wire at a distance r from the axis of the wire. Draw your Amperian loop on the diagram to the right.



B) Find the magnetic flux Φ_B through the rectangular surface shown, with length x and width R. Draw your dA on the diagram, and label its parameters.

C) Find the inductance per unit length of the wire. (Hint: How many "turns" are there?)

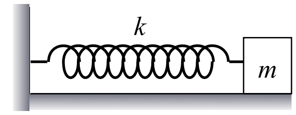
D) Find the magnetic energy density u_B at a distance r from the axis of the wire.

E) Consider a cylindrical shell of radius $r < R$, length x, and thickness dr. Find the differential energy dU stored in the shell.

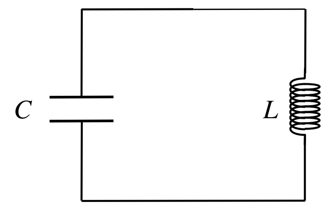
F) Integrate to find the total energy per unit length stored in the wire.

G) Use the expression for the energy stored in an inductor to find the inductance per unit length of the wire, and compare with part C).

LC Circuits



When an ideal mass-spring system oscillates, a fixed total energy is exchanged between kinetic energy in the mass and elastic potential energy in the spring. The electrical analogue of this is an *LC circuit*, consisting of an inductor and a capacitor. The capacitor behaves much like the spring, storing potential energy, and the inductor behaves like the mass, providing "inertia" by resisting changes in current. If there is no loss due to resistance, the total energy U_{LC} of an *LC* circuit is partly in the inductor and partly in the capacitor. Imagine that at a certain instant the capacitor has a charge q and a current i flows in the circuit.



A) Write the expression for U_{LC} at this instant:

B) Take the time derivative to find $\dfrac{dU_{LC}}{dt}$:

C) If there is no loss of energy, what is $\dfrac{dU_{LC}}{dt}$?

D) Write a second-order differential equation in q, similar to $\dfrac{d^2x}{dt^2} = -\dfrac{k}{m}x$ for the mass-spring system:

The solution of $\dfrac{d^2x}{dt^2} = -\dfrac{k}{m}x$ from Simple Harmonic Motion is: $x = x_\mathrm{m}\cos(\omega t + \phi)$ where $\omega = \sqrt{\dfrac{k}{m}}$. The period in SHM is given by $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{m}{k}}$, and the frequency is $f = \dfrac{1}{T} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$.

E) Write the corresponding solution of the *LC* equation, and find the period and frequency of the *LC* circuit:

$q =$ where

$T =$ and $f =$

Maxwell's Equations

The four equations that we have developed this semester are together called *Maxwell's Equations*, summarized to the right.

There is a more general form of Ampere's Law which you'll learn if you take another course in E & M. You might guess that it contains the expression $\dfrac{d\Phi_E}{dt}$, so that it's more or less symmetric with Faraday's Law.

1.	Gauss' Law for Electricity	$\varepsilon_\mathrm{o}\oint \vec{E}\bullet d\vec{A} = q_\mathrm{in}$
2.	Gauss' Law for Magnetism	$\oint \vec{B}\bullet d\vec{A} = 0$
3.	Ampere's Law	$\oint \vec{B}\bullet d\vec{s} = \mu_\mathrm{o} i_\mathrm{in}$
4.	Faraday's Law of Induction	$\oint \vec{E}\bullet d\vec{s} = -\dfrac{d\Phi_B}{dt}$

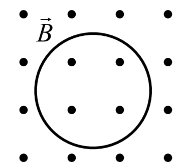
Maxwell's Equations form the basis for a complete theory of *Electromagnetism* the way Newton's laws form a basis for classical mechanics.

AP Physics C
Unit 16 Practice Test

Name ___________________________________

Part I
Multiple Choice

1. A magnetic field perpendicular to the plane of a wire loop is uniform in space but changes with time t in the region of the loop. If the induced emf in the loop increases linearly with time t, then the magnitude of the magnetic field must be proportional to

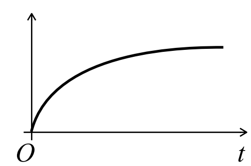


A) t^3
B) t^2
C) t
D) t^0 (i.e., constant)
E) $t^{1/2}$

For questions 2 - 4: A circuit consists of a resistor R, an inductor L, and an open switch S connected in series with a battery. The switch is then closed at time $t = 0$.

2. If the current in the circuit is i at time t, what energy is stored in the circuit in addition to that stored in the battery?
A) Li
B) i^2R
C) $\frac{1}{2}Li^2$
D) $Li + i^2R$
E) $\frac{1}{2}Li^2 + i^2R$

3. Which of the following quantities could be represented as a function of time by the graph shown to the right?



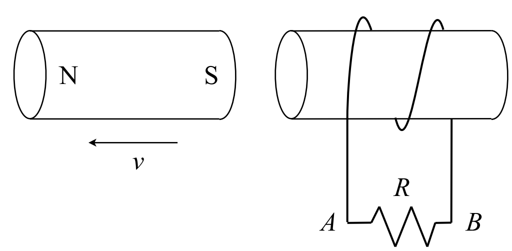
I. The potential difference across the resistor
II. The potential difference across the inductor
III. The current in the circuit

A) I only B) II only C) I, II, and III D) II and III only E) I and III only

4. The change in current when the switch is closed is determined by the inductive time constant τ_L. If the inductance is doubled and the resistance is halved, the new inductive time constant is

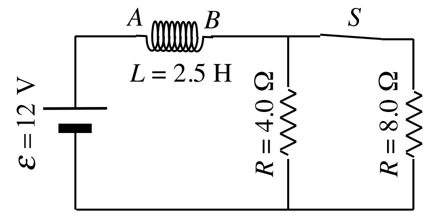
A) $\frac{1}{4}\tau_L$
B) $\frac{1}{2}\tau_L$
C) τ_L
D) $2\tau_L$
E) $4\tau_L$

5. A strong bar magnet is held very close to a solenoid as shown in the diagram. As the magnet is moved away from the solenoid at constant speed, what is the direction of conventional current through the resistor shown, and what is the direction of the force on the magnet because of the induced current?



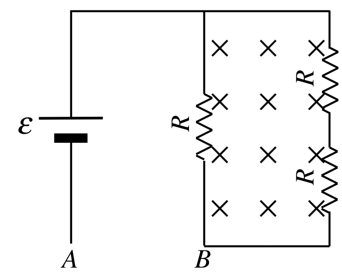
	Current through resistor	Force on magnet
A)	From A to B	To the left
B)	From B to A	To the left
C)	From A to B	To the right
D)	From B to A	To the right
E)	No current	To the right

6. The circuit shown to the right has been operating for a long time with switch S *closed*. The instant after the switch is *opened*, what is the inductive emf $\mathcal{E}_L$ of the inductor, and which side of the inductor is at the higher potential?



	Inductive emf	Higher potential
A)	30 V	(equal)
B)	12 V	*A*
C)	12 V	*B*
D)	6 V	*A*
E)	6 V	*B*

7. A magnetic field is directed into the page in the right side of the circuit shown to the right. The magnetic field is weakening, which induces a constant emf of magnitude $\mathcal{E}$ around that side of the circuit. A battery of emf $\mathcal{E}$ is connected only at its positive terminal as shown. The potential difference between points A and B is



A) $\frac{4}{3}\mathcal{E}$

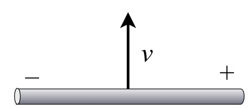
B) $\frac{2}{3}\mathcal{E}$

C) $\frac{1}{3}\mathcal{E}$

D) $\mathcal{E}$

E) 0

For questions 8 and 9: A wire moves upward through a magnetic field so that an emf is induced between its ends. The signs of the induced charges are shown.



8. The magnitude of the induced emf is

I. Directly proportional to the strength of the magnetic field
II. Directly proportional to the velocity of the wire
III. Directly proportional to the diameter of the wire

A) I only B) II only C) I, II, and III D) II and III only E) I and II only

9. The direction of the magnetic field could be

A) into the page
B) out of the page
C) towards the bottom of the page
D) towards the top of the page
E) towards the right

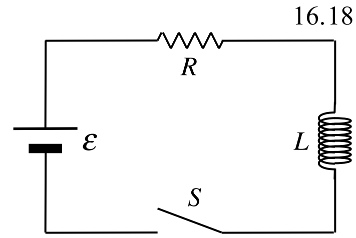
10. A solenoid is constructed with N turns wrapped around a core of radius R and length ℓ. If the current through the solenoid is cut in half, the inductance of the solenoid is

A) unchanged
B) quartered
C) halved
D) doubled
E) quadrupled

11. The numeric value of the quantity $\sqrt{\dfrac{\mu_o}{\varepsilon_o}}$ is 120π. The unit of this quantity is

A) Farads
B) Joules
C) Volts
D) Ohms
E) Henrys

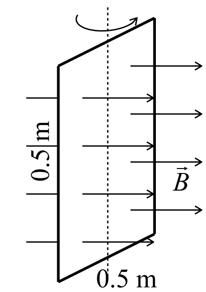
12. For the *LR* circuit shown to the right, which of the following changes would double the initial voltage across the inductor (at the instant the switch is closed)?



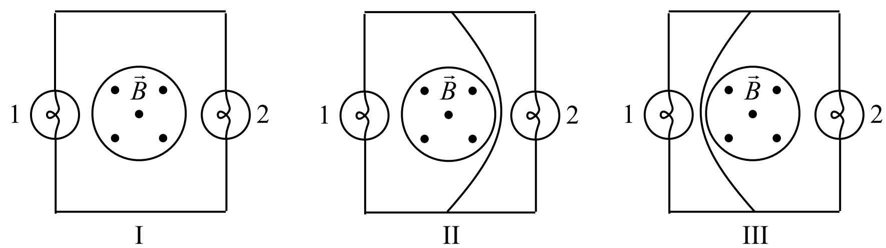
    I. Reduce the resistance to *R*/2
    II. Double the inductance of the inductor to 2*L*
    III. Add a second identical inductor to the circuit in parallel with the first

A) I only
B) II only
C) III only
D) I and II together
E) none of these

13. A square loop is placed in a uniform magnetic field perpendicular to the plane of the loop as shown to the right. The loop is 0.5 m on a side, and the magnetic field has a strength of 2.0 T. If the loop is rotated through an angle of 90° in 0.1 second, the average emf induced in the loop is



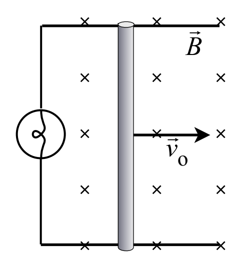
A) 0.025 V
B) 0.40 V
C) 5.0 V
D) 10 V
E) 80 V

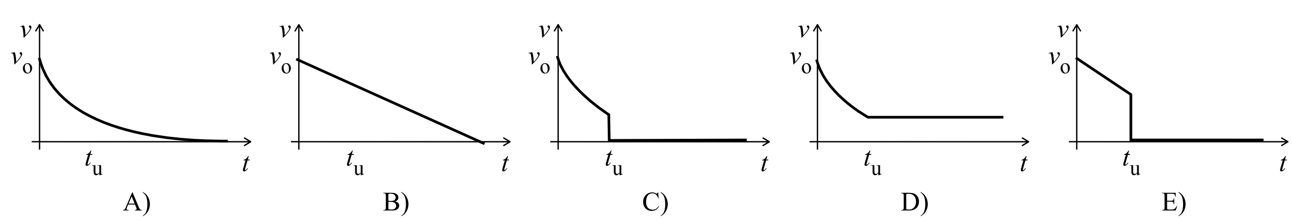


14. In each of the three circuits above, the bulbs are identical. A solenoid is perpendicular to the plane of the page with its magnetic field out of the page as shown. The magnetic field of the solenoid is reduced at the same constant rate in each circuit. In circuits II and III, there is an additional wire of negligible resistance. The rank of the circuits, in order of the brightness of bulb 1, from brightest to dimmest is

A) II > I > III
B) I = II = III
C) I > II = III
D) II = III > I
E) I > II > III

15. A wire is free to slide on rails as shown to the right. A bulb is connected between the rails, and a uniform magnetic field is directed into the page. Neglect friction and gravitational forces. The wire is given an initial speed v_o at $t = 0$ and released. At $t = t_u$, the bulb is unscrewed from its socket. The graph that best represents the speed of the wire as a function of time is

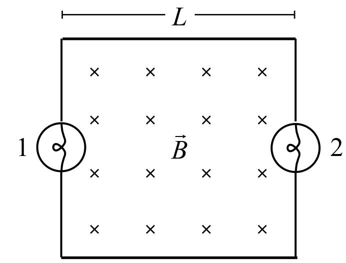




1.B 2.C 3.E 4.E 5.D 6.E 7.A 8.E 9.B 10.A 11.D 12.E 13.C 14.A 15.D

Part II
Show your work
Credit depends on the quality and clarity of your explanations

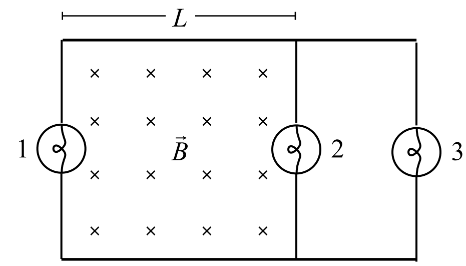
1. A square conducting loop of side L contains two identical lightbulbs, 1 and 2, as shown to the right in circuit I. There is a magnetic field directed into the page in the region inside the loop with magnitude as a function of time t given by $B(t) = at + b$, where a and b are positive constants. The lightbulbs each have constant resistance R_o. Express all answers in terms of the given quantities and fundamental constants.



Circuit I

A) Derive an expression for the magnitude of the emf generated in the loop.
B) i. Determine an expression for the current through bulb 2.
   ii. Indicate on the diagram to the right the direction of the current through bulb 2.
C) Derive an expression for the power dissipated in bulb 1.

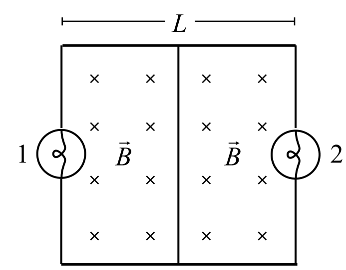
Another identical bulb 3 is now connected in parallel with bulb 2, but it is entirely outside the magnetic field, as shown in circuit II.



Circuit II

D) Compared with circuit I, is bulb 1 now *brighter*, *dimmer*, or *the same brightness*? Justify your answer.

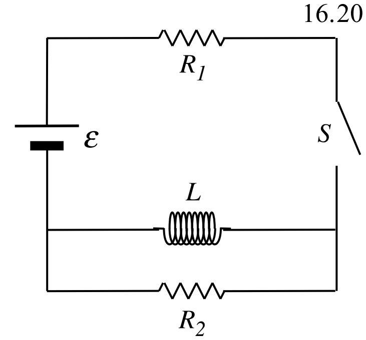
Now the portion of the circuit containing bulb 3 is removed, and a wire is added to connect the midpoints of the top and bottom of the original loop, as shown in circuit III.



Circuit III

E) Compared with circuit I, is bulb 1 now *brighter*, *dimmer*, or *the same brightness*? Justify your answer.

2. In the circuit shown to the right, two resistors R_1 and R_2 and an inductor of inductance L are connected to a battery of emf $\mathcal{E}$ and a switch S. The switch is closed at time $t = 0$. Express all algebraic answers in terms of the given quantities and fundamental constants.



A) Determine the current through each resistor immediately after the switch is closed.

B) Determine the magnitude of the initial rate of change of current $\frac{di}{dt}$ in the inductor.

C) Determine the current through the battery a long time after the switch has been closed.

D) On the axes to the right, sketch a graph of the current through the battery as a function of time.

Some time after steady state has been reached, the switch is opened.

E) Determine the voltage across each resistor just after the switch is opened.